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de Mécanique des Fluides Numérique

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Hydrodynamique à surface libre

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École organisée à l'initiative du Réseau MFN
avec le soutien de la formation permanente du CNRS

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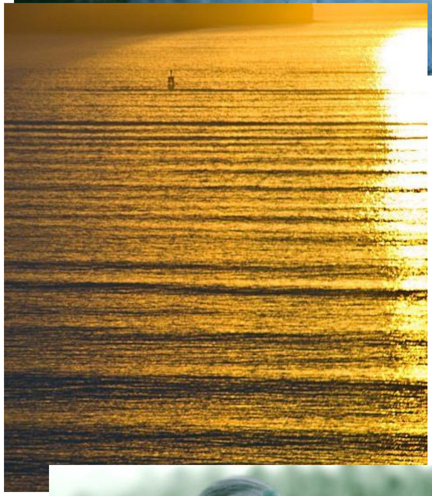
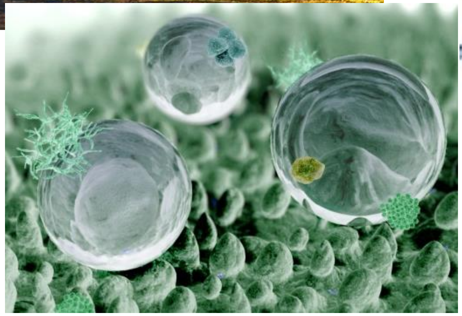
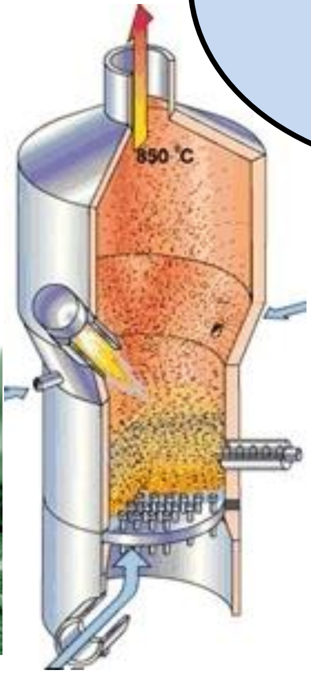
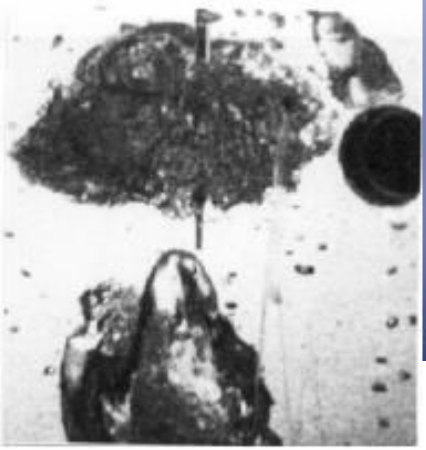
Hydrodynamique à surface libre : quelle classe de problèmes ?





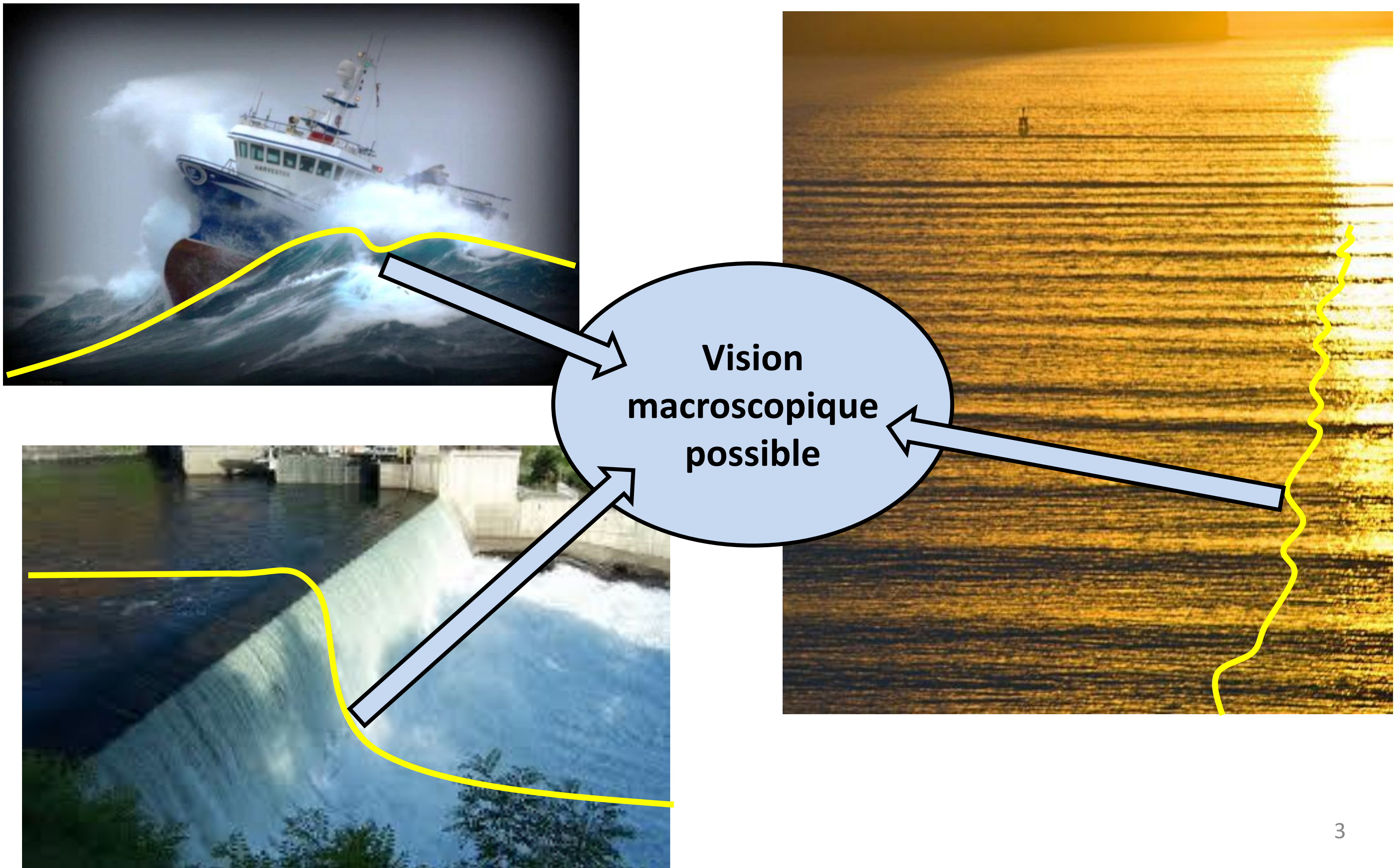
Ecoulements diphasiques à phases séparées



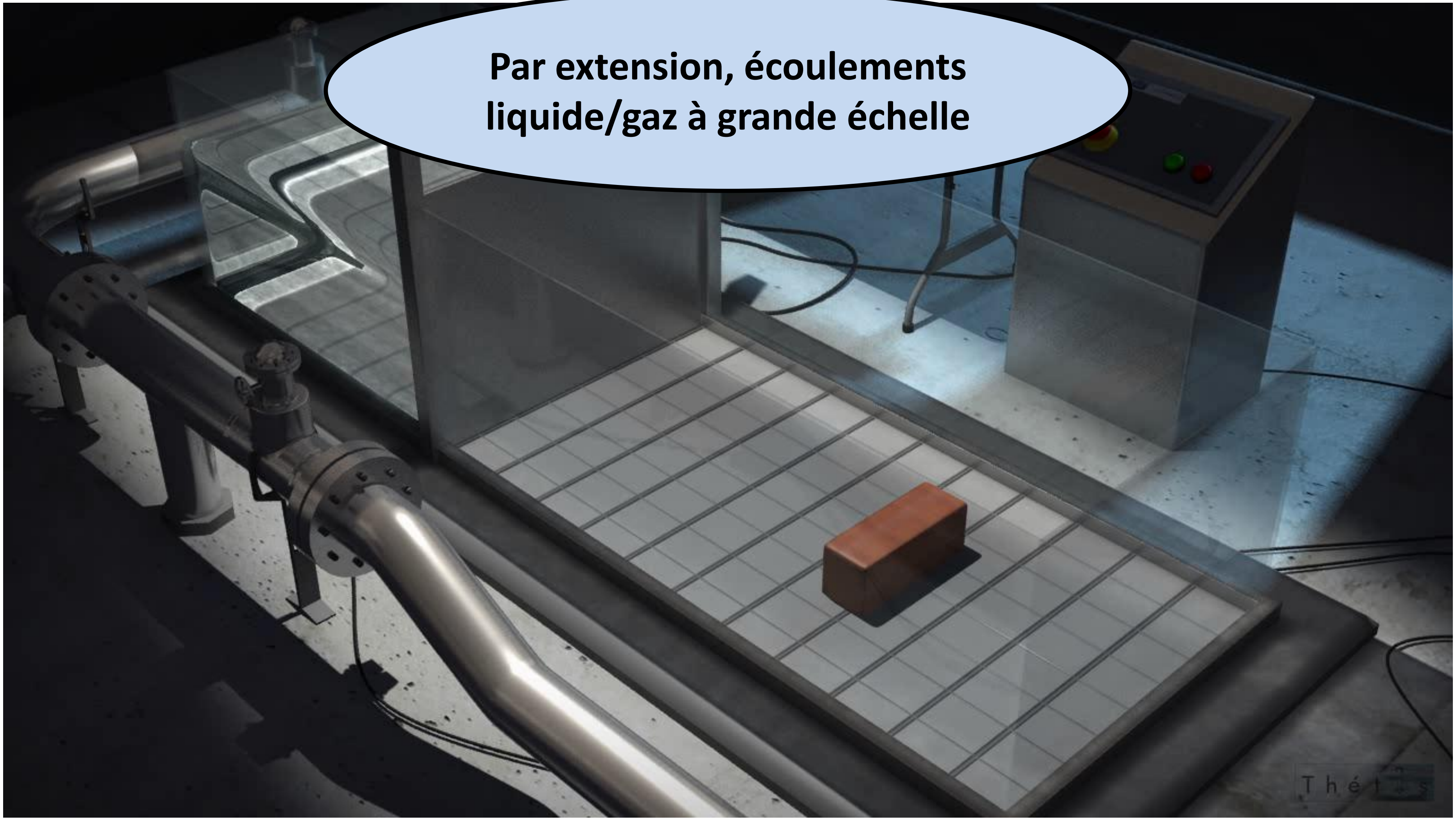

Sources google image

Hydrodynamique à surface libre : quelle classe de problèmes ?



Hydrodynamique à surface libre : quelle classe de problèmes ?

Par extension, écoulements liquide/gaz à grande échelle



[1] Vincent et al. JCP 2010

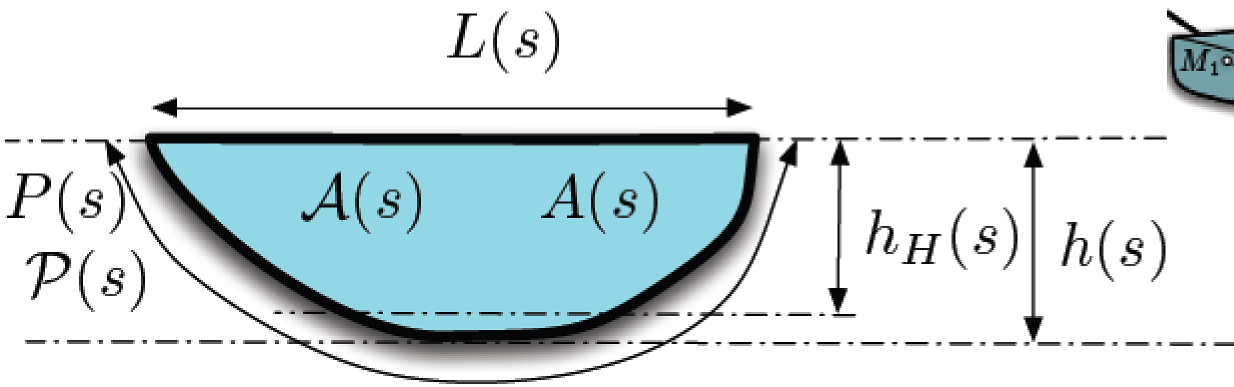
Plan du cours

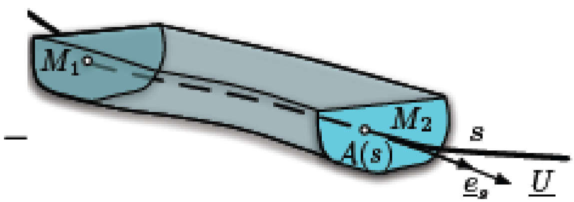
I. Hydrodynamique des écoulements à surface libre

I.1 Concepts généraux et définitions

- Section : A
- Périmètre mouillé : P
- Largeur mirroir : L

- Rayon hydraulique : $R_H = A/P$
- Diamètre hydraulique : $D_H = 4\ R_H$
- Hauteur hydraulique : $h_H = A/L$





- On suppose souvent $h_H \sim h$
- Section rectangulaire avec $L \gg h$: $R_H \sim h$
- Section en demi-cercle de rayon R : $R_H = R/2$

[2] Thual Cours 2011

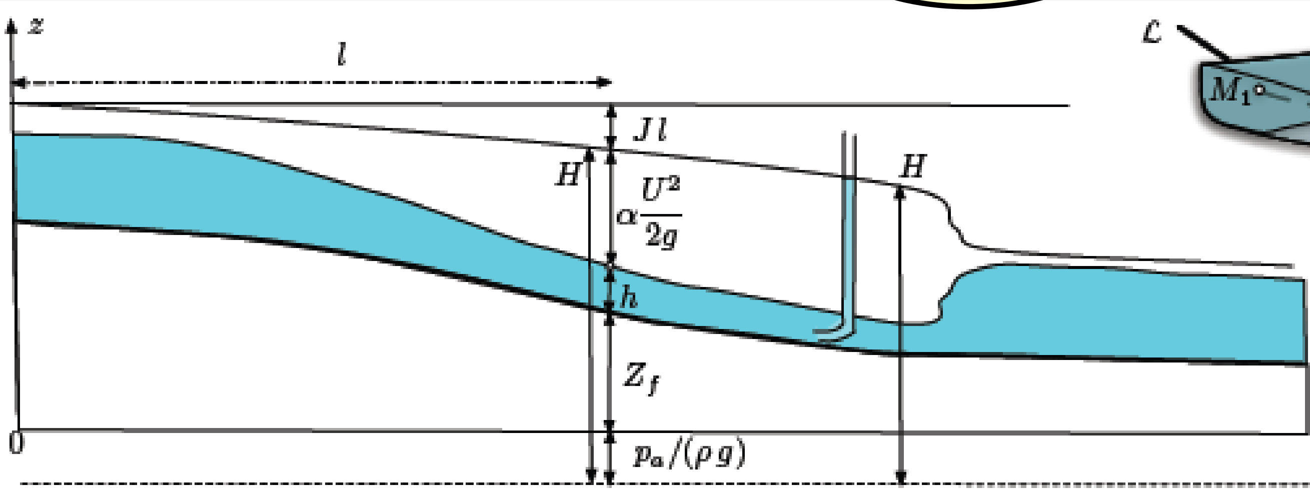
I. Hydrodynamique des écoulements à surface libre

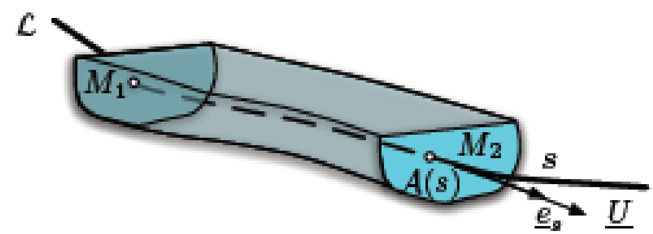
I.1 Concepts généraux et définitions

$$\int_{\mathcal{L}} \underline{\text{grad}}\, H \cdot \underline{dM} = \frac{1}{g} \int_{\mathcal{L}} \left(-\frac{\partial \underline{U}}{\partial t} + \nu \Delta \underline{U} - \underline{\text{div}}\, \underline{\underline{R}} \right) \cdot \underline{dM}$$

Charge hydraulique des écoulements à surface libre :

$$H(s) = \frac{p_a}{\rho g} + Z_f(s) + h(s) + \alpha(s) \frac{U^2(s)}{2g}$$





[2] Thual Cours 2011

I. Hydrodynamique des écoulements à surface libre

I.1 Concepts généraux et définitions

Le coefficient de Coriolis α est en général compris entre 1 (écoulement uniforme) et 1.2 (rivières).

La perte de charge linéique dans un canal est $J=-dH/dl=-dZ_f/dl$ si p_a, U et h sont constants.

Pour connaitre la charge, il reste à déterminer a priori (sans résoudre les équations de conservation) la vitesse moyenne U sur s :

loi de Chezy $$U = C\sqrt{Ri}$$

formule de Manning-Strickler $$C = KR^{1/6}$$

$$U = KR^{2/3} i^{1/2}$$

K=coefficient de rugosité

K=10 $m^{1/3}$.s pour des berges végétalisées à K=75 $m^{1/3}$.s pour du béton

i=pente au fond du tronçon de cours d'eau (supposée constante)

R=rayon hydraulique

[3] Degoutte Cours 2014

I. Hydrodynamique des écoulements à surface libre

I.1 Concepts généraux et définitions

Typologie des écoulements à surface libre : fluvial / torrentiel

Charge spécifique moyenne $$H_s = h + \alpha \frac{U^2}{2g}$$

Energie spécifique $$\frac{dH_s}{dh} = 1 - \alpha \frac{U^2}{2gh}$$

Energie spécifique minimale $$h_c = \alpha \frac{U_c^{\,2}}{2g}$$

Nombre de Froude : son carré correspond au rapport entre l'énergie cinétique du fluide en mouvement et l'énergie potentielle induite par la pesanteur

$$Fr = \frac{U}{\sqrt{gh}} \qquad \left\{ \begin{array}{l} H_s = h\left(1 + \dfrac{\alpha}{2} Fr^2\right) \\[2ex] \dfrac{dH_s}{dh} = 1 - Fr^2 \end{array} \right.$$

Fr=1 est une valeur critique

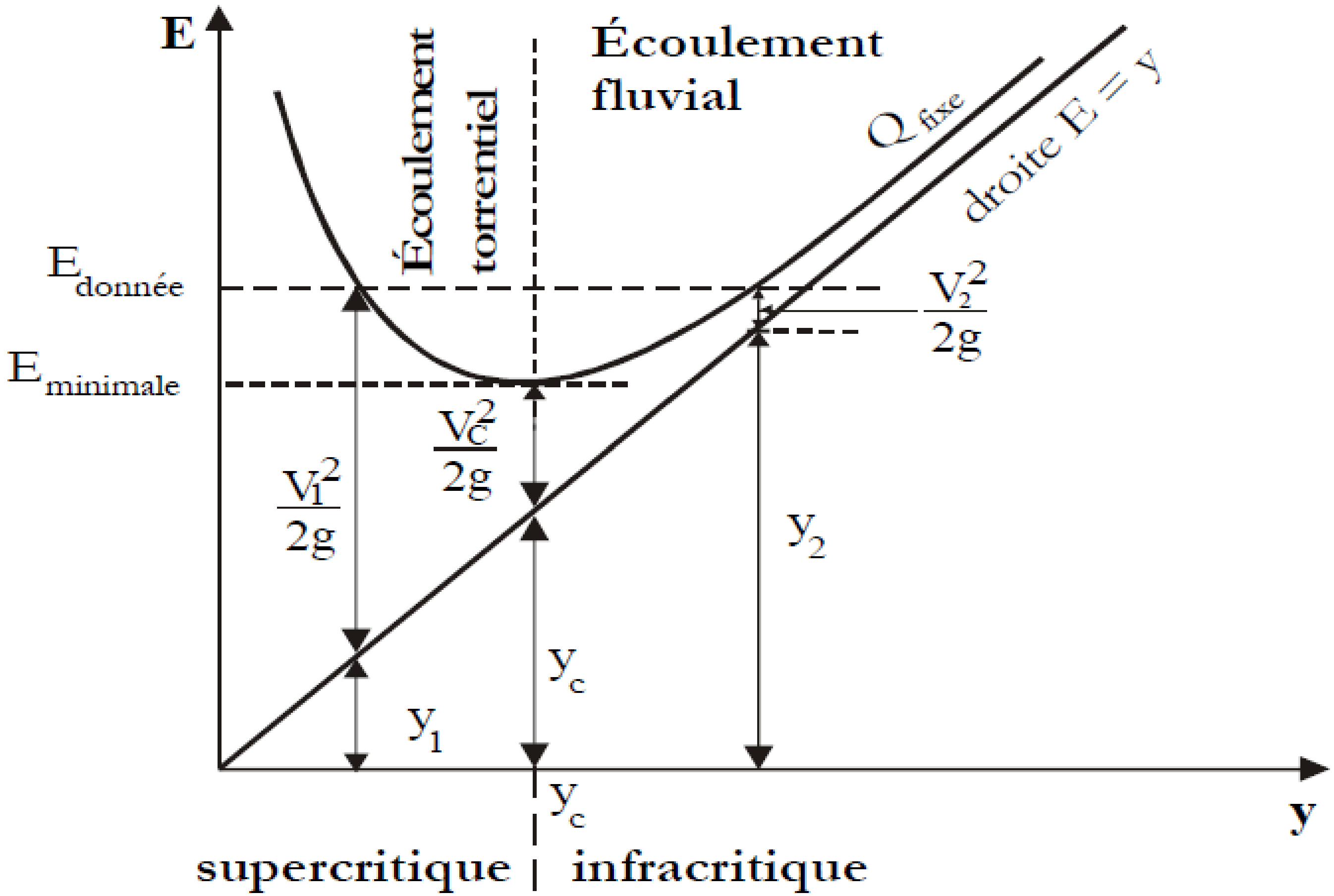
Charge moyenne minimale

[3] Degoutte Cours 2014

I. Hydrodynamique des écoulements à surface libre

I.1 Concepts généraux et définitions

Typologie des écoulements à surface libre : fluvial / torrentiel

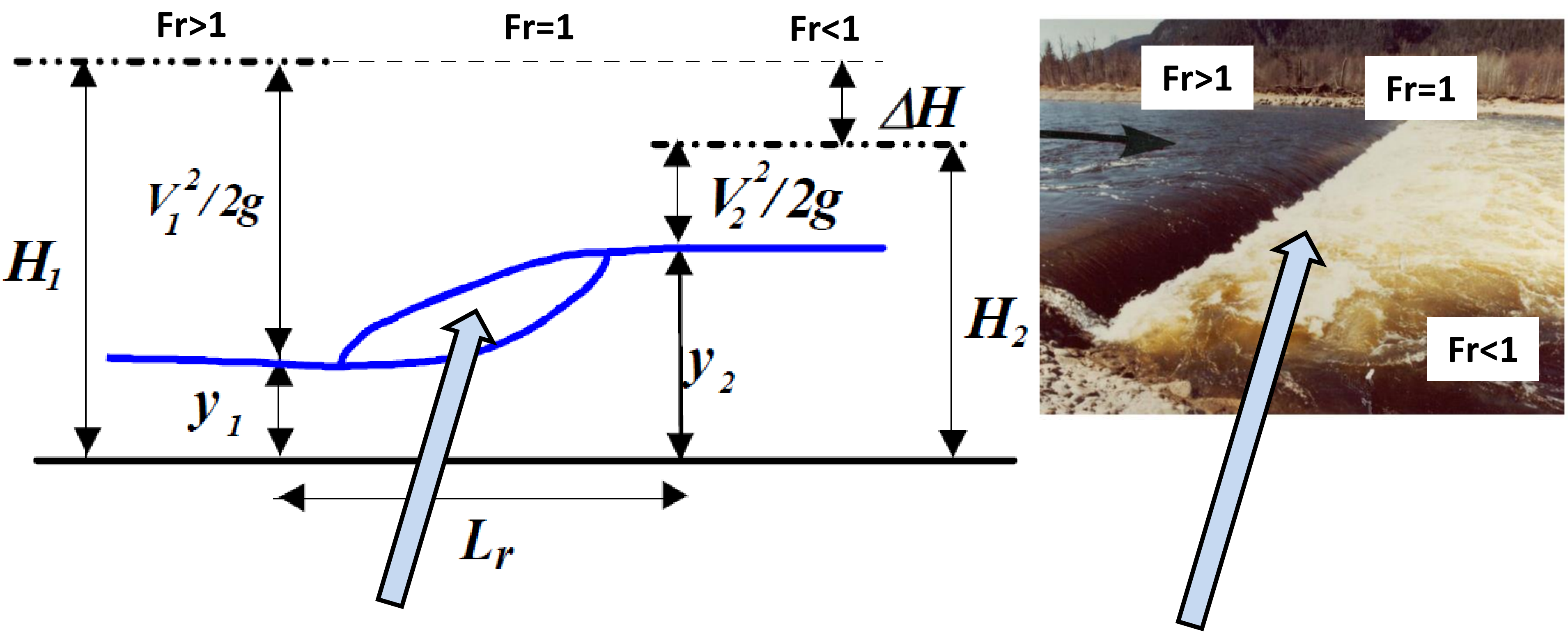


[4] S. Bennis, Cours 2015

I. Hydrodynamique des écoulements à surface libre

I.1 Concepts généraux et définitions

Ressaut hydraulique : torrentiel vers fluvial



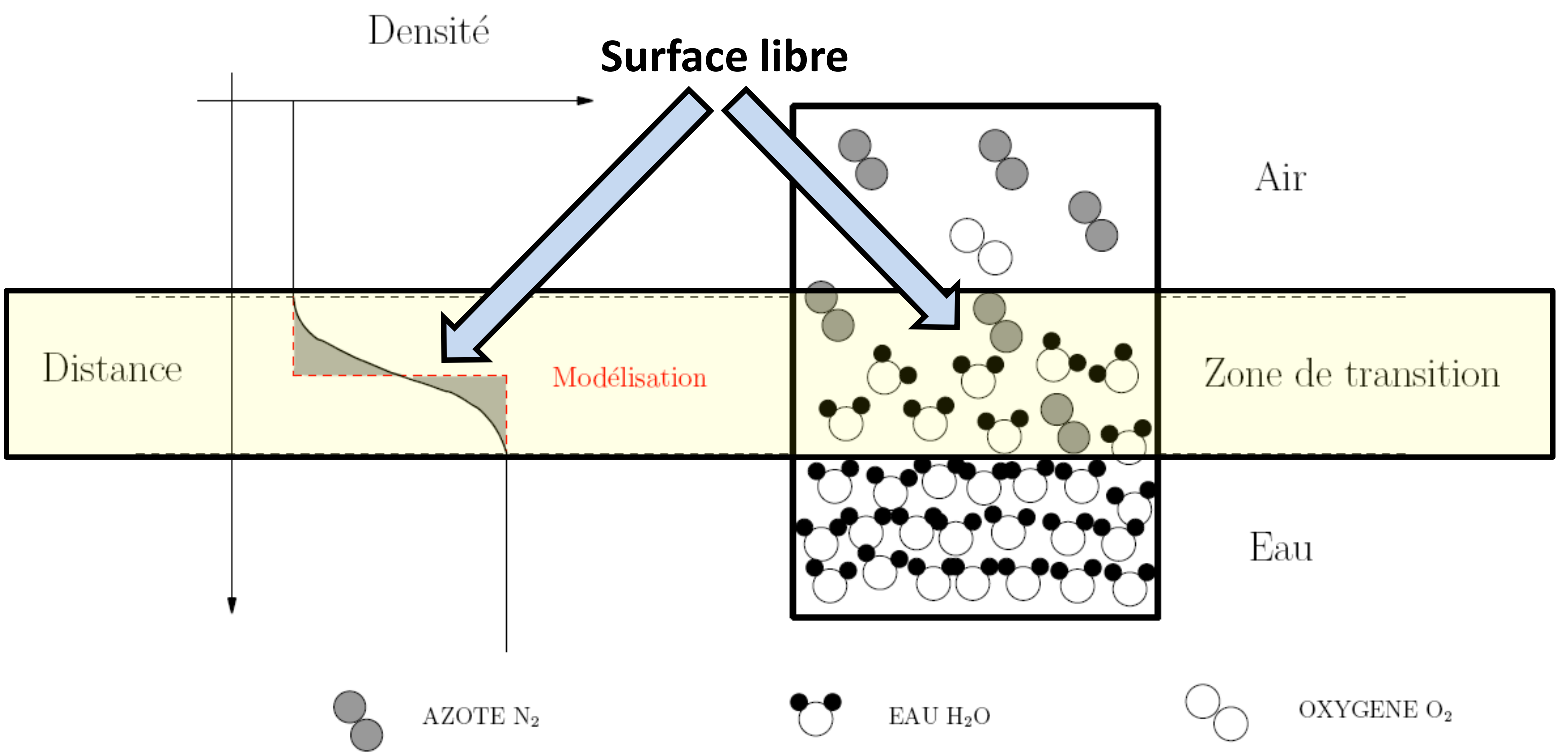
Forte discontinuité de la charge moyenne, tourbillons, remous, entrainement de bulles d'air
Importante agitation qui dissipe l'énergie acquise dans le tronçon torrentiel

[3] Degoutte Cours 2014

I. Hydrodynamique des écoulements à surface libre

I.1 Concepts généraux et définitions

Différentes échelles d'une surface libre : échelle moléculaire / milieu continu



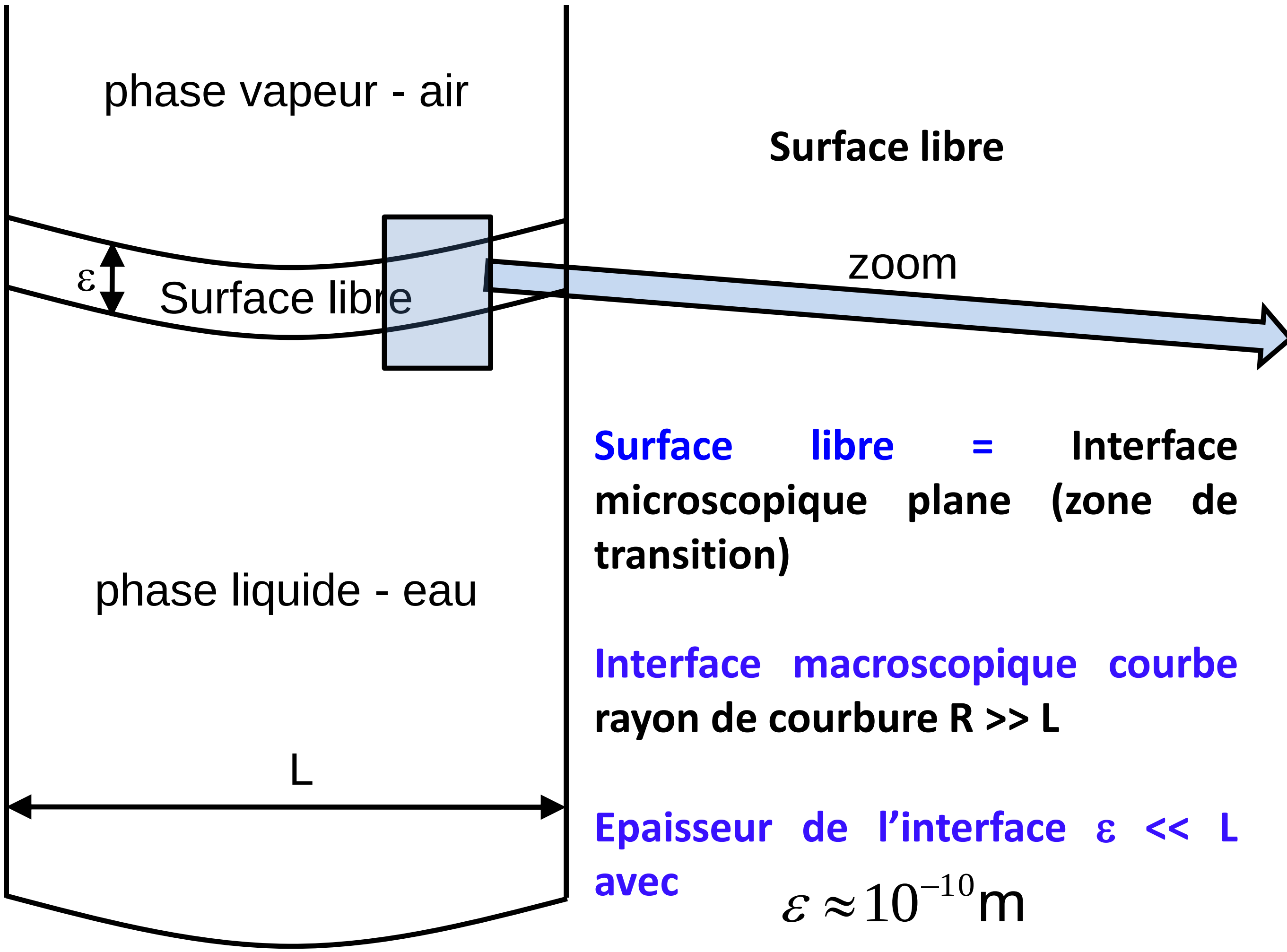
Représentation milieu continu (approche discontinue de Gibbs)

Représentation échelle moléculaire

[5] J. Larocque, Thèse 2015

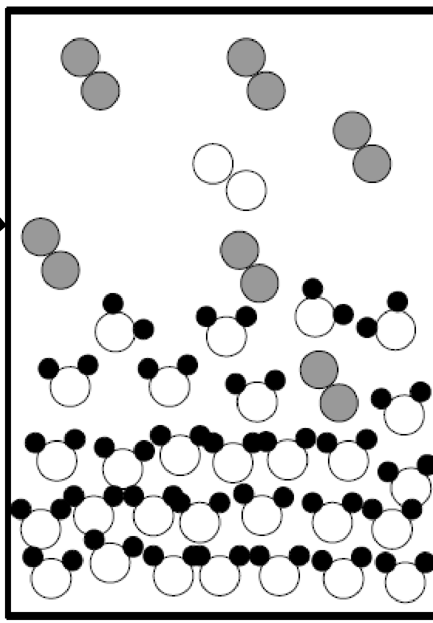
I. Hydrodynamique des écoulements à surface libre

I.1 Concepts généraux et définitions



Vue macroscopique

Surface libre



Volume élémentaire représentatif (VER)

Surface libre = Interface microscopique plane (zone de transition)

Interface macroscopique courbe rayon de courbure R >> L

Epaisseur de l'interface ε << L avec $\varepsilon \approx 10^{-10}\text{m}$

[5] J. Larocque, Thèse 2015

I. Hydrodynamique des écoulements à surface libre

I.1 Concepts généraux et définitions

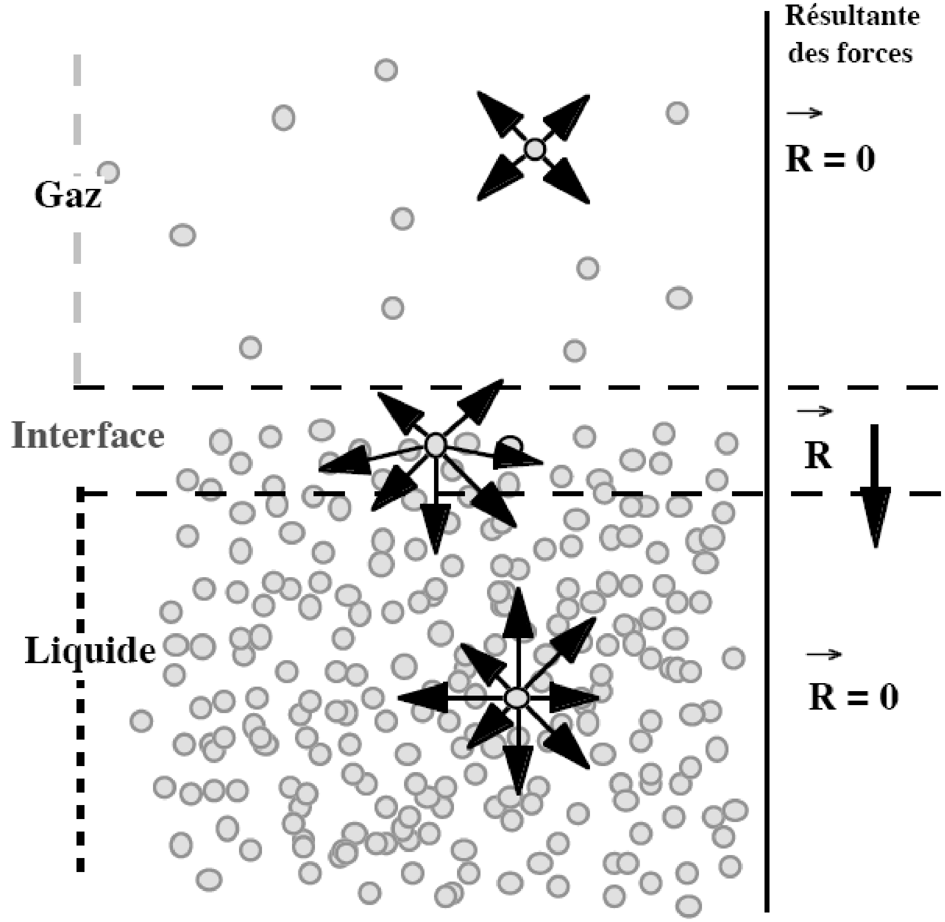
Phénomènes dus aux *interfaces* du type liquide-vapeur, liquide-solide et solide-vapeur.

Echelle macroscopique: pas de mouvements d'ensemble du liquide.

Echelle microscopique: molécules situées à l'*interface*: subissent une force résultante dirigée vers le liquide

La couche superficielle d'un liquide est soumise à une force qui tend à *réduire* cette surface

Tensions superficielles : vision microscopique

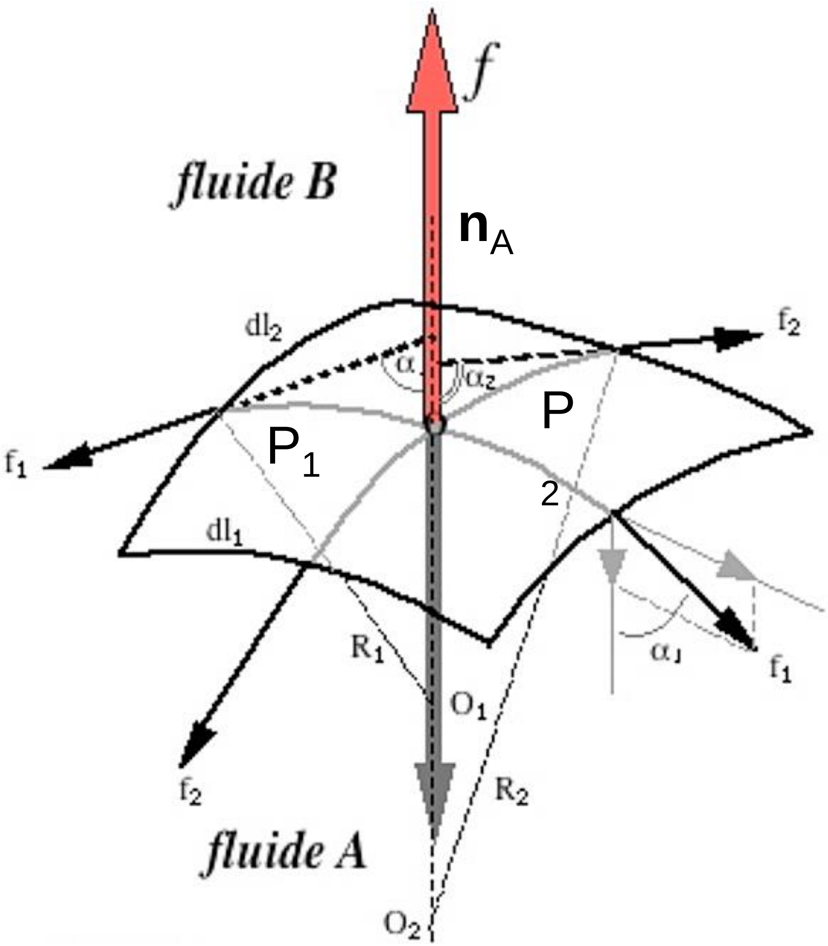


[5] J. Larocque, Thèse 2015

I. Hydrodynamique des écoulements à surface libre

I.1 Concepts généraux et définitions

Tensions superficielles : vision macroscopique



Loi de Laplace-Young

$$\Delta p = 2\sigma \frac{\cos\alpha_1}{dl_1} + 2\sigma \frac{\cos\alpha_2}{dl_2}$$

$$= \sigma\left(2\frac{\cos\alpha_1}{dl_1} + 2\frac{\cos\alpha_2}{dl_2}\right)$$

$$= \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

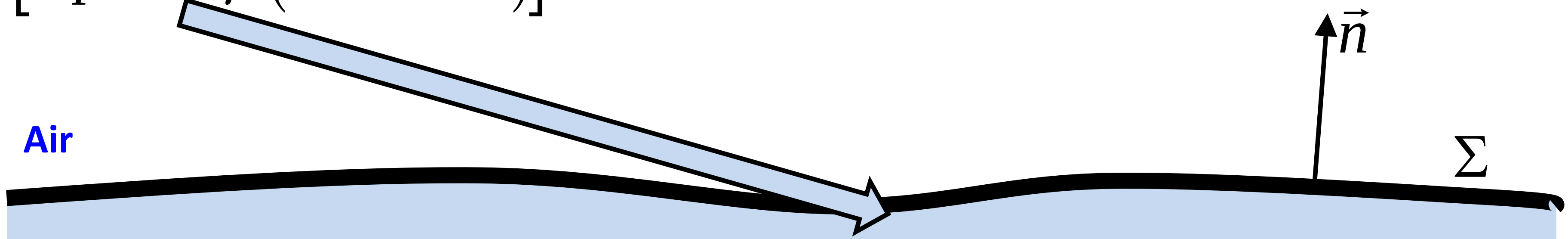
I. Hydrodynamique des écoulements à surface libre

I.1 Concepts généraux et définitions

Définition mathématique et macroscopique d'une surface libre : conditions de surface libre

$$\vec{u}\cdot\vec{n}=0$$
$$\left[-p\overline{\overline{Id}}+\mu\left(\nabla\vec{u}+\nabla^{t}\vec{u}\right)\right]\cdot\vec{n}=\sigma\kappa\vec{n}$$

$$\Rightarrow\quad \vec{u}\cdot\vec{n}=0$$
$$p=p_{atm}$$



Hypothèses :

Interface macroscopique (κ=1/R<<H) $\quad \Delta p=\sigma/R<<p-p_{atm}=\rho gH$

Couche limite air/eau négligeable $\quad \delta\approx C\,\mathrm{Re}^{-1/2}<<H \Rightarrow \mathrm{Re}>>1$

Pression constante dans l'air $\quad p_{air}=p_{atm} \qquad p_{\Sigma}=p_{atm}$

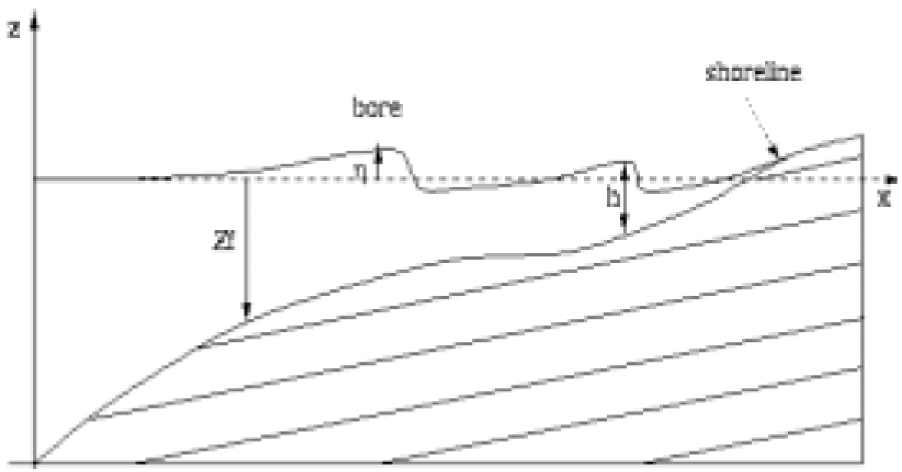
I. Hydrodynamique des écoulements à surface libre

I.2 Modélisations

I.2.1 Approches eulériennes intégrées macroscopiques – Equations de Saint-Venant (SWE)

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S}$$

$$\text{where } \mathbf{q} = \begin{pmatrix} h \\ hu \end{pmatrix}, \mathbf{F} = \begin{pmatrix} hu \\ hu^2 + \frac{g}{2}h^2 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 0 \\ -gh\frac{\partial Zf}{\partial x} \end{pmatrix}$$



Vision intégrée :

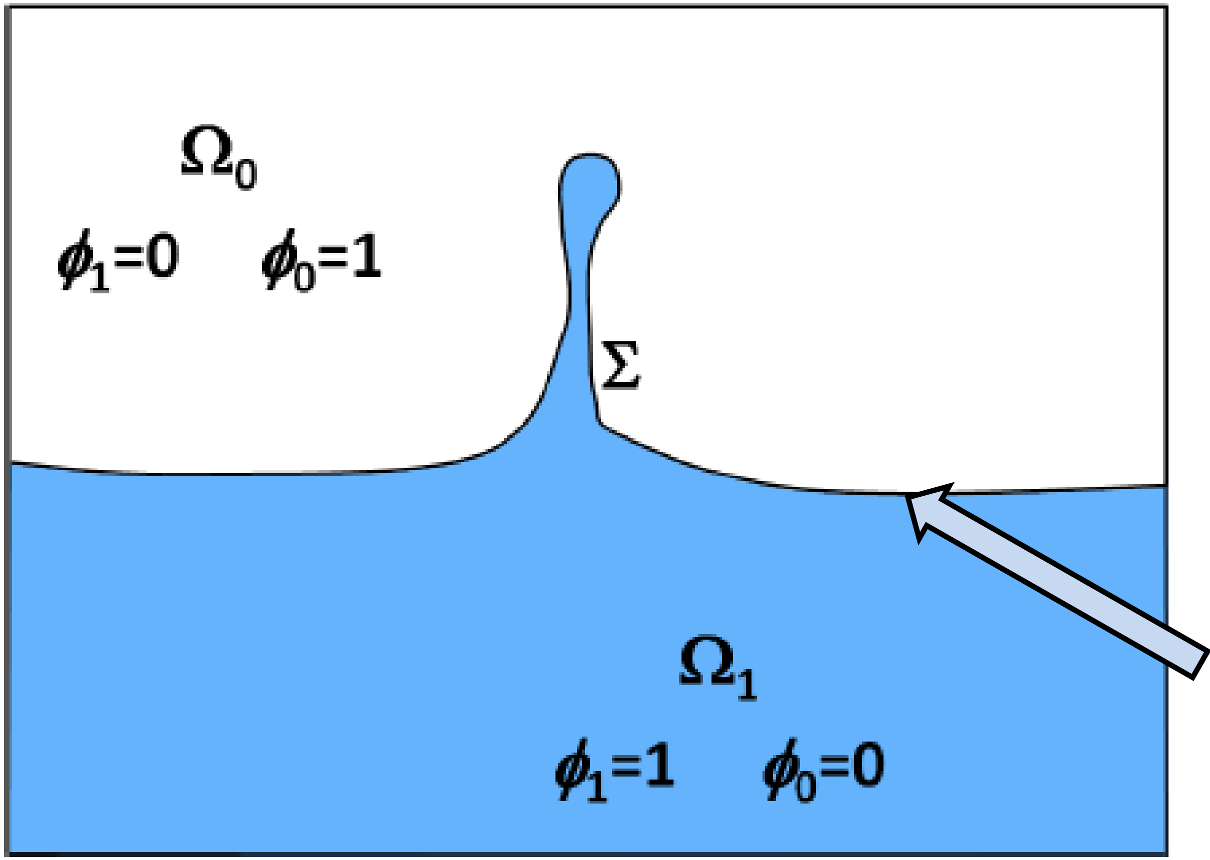
Air négligeable *a priori* (termes sources de vent) à P=P_{atm}
Pression hydrostatique dans l'eau
Vitesses moyennes débitantes

[6] S. Vincent, Thèse 1999 *[7] Vincent et al., JHR, 2001*

I. Hydrodynamique des écoulements à surface libre

I.2 Modélisations

I.2.2 Approches eulériennes résolues microscopiques– Modèle 1-fluide (MUF)



Vision approche discontinue de Gibbs :

Air (milieu 0)
Eau (milieu 1)

Introduction de fonctions indicatrices de phase

L'interface Σ (surface libre) est une surface de contact entre deux fluides

[8] S. Vincent, HDR, 2010

I. Hydrodynamique des écoulements à surface libre

I.2 Modélisations

I.2.2 Approches eulériennes résolues microscopiques– Modèle 1-fluide (MUF)

$$\tilde{\rho}\left(\frac{\partial \tilde{\mathbf{u}}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \tilde{\mathbf{u}}\right) = -\nabla \tilde{p} + \rho \mathbf{g} + \nabla \cdot \left(\tilde{\mu}[\nabla \tilde{\mathbf{u}} + \nabla^T \tilde{\mathbf{u}}]\right) + \mathbf{F}_{TS}$$

$$\nabla \cdot \tilde{\mathbf{u}} = 0$$

$$\frac{\partial \widetilde{C}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \widetilde{C} = 0$$

Vision filtrée (moyennée) de l'approche discontinue de Gibbs :

Air (milieu 0)
Eau (milieu 1)

$$\widetilde{\phi}(M,t) = \frac{1}{mes(V_{er})} \int_{V_{er}} G \circ \phi(M,t)\, dV$$

Introduction de fonctions indicatrices de phase moyennées (fraction volumiques)

$$\mathbf{u} = C\mathbf{u}_1 + (1-C)\mathbf{u}_0$$

$$p = Cp_1 + (1-C)p_0$$

$$\boldsymbol{F}_{TS}(\widetilde{C}) = \sigma \kappa n \delta_i = \sigma \nabla \cdot \left(\frac{\nabla \widetilde{C}}{\|\nabla \widetilde{C}\|}\right) \nabla \widetilde{C}$$

L'interface (surface libre) n'est pas connue, elle est reconstruite à partir du description volumique (moyennée) du milieu diphasique (mélange)

[8] S. Vincent, HDR, 2010 *[9] Kataoka, IJMF, 1986*

I. Hydrodynamique des écoulements à surface libre

I.2 Modélisations

I.2.2 Approches eulériennes résolues microscopiques– Modèle 1-fluide (MUF)

$$\tilde{\rho}\left(\frac{\partial \tilde{\mathbf{u}}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \tilde{\mathbf{u}}\right) = -\nabla \tilde{p} + \rho \mathbf{g} + \nabla \cdot \left(\tilde{\mu}[\nabla \tilde{\mathbf{u}} + \nabla^T \tilde{\mathbf{u}}]\right) + \mathbf{F}_{TS}$$

$$\nabla \cdot \tilde{\mathbf{u}} = 0$$

$$\frac{\partial \tilde{C}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \tilde{C} = 0$$

Vision filtrée (moyennée) de l'approche discontinue de Gibbs :

$$\tilde{\rho} = \rho_1 \tilde{C} + \rho_0 (1 - \tilde{C})$$

$$\tilde{\mu} = \mu_1 \tilde{C} + \mu_0 (1 - \tilde{C})$$

Une reconstruction du milieu diphasique est nécessaire

Modèle complètement non-linéaire

$$\frac{\partial \tilde{\rho}}{\partial t} + \nabla \cdot (\tilde{\rho}\tilde{\mathbf{u}}) = \frac{\partial \tilde{\rho}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \tilde{\rho} = (\rho_1 - \rho_0)\left[\frac{\partial \tilde{C}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \tilde{C}\right] = 0$$

[8] S. Vincent, HDR, 2010   *[9] Kataoka, IJMF, 1986*

I. Hydrodynamique des écoulements à surface libre

I.2 Modélisations

I.2.3 Approches lagrangiennes résolues microscopiques (DNS)

$$\rho_0\left(\frac{\partial \mathbf{u}_0}{\partial t}+\mathbf{u}_0\cdot\nabla\mathbf{u}_0\right)=-\nabla p_0+\rho_0\mathbf{g}+\nabla\cdot\left(\mu_0[\nabla\mathbf{u}_0+\nabla^T\mathbf{u}_0]\right)$$

$$\nabla\cdot\mathbf{u}_0=0$$

dans la phase 0

$$\rho_1\left(\frac{\partial \mathbf{u}_1}{\partial t}+\mathbf{u}_1\cdot\nabla\mathbf{u}_1\right)=-\nabla p_1+\rho_1\mathbf{g}+\nabla\cdot\left(\mu_1[\nabla\mathbf{u}_1+\nabla^T\mathbf{u}_1]\right)$$

$$\nabla\cdot\mathbf{u}_1=0$$

dans la phase 1 et

$$\left[p_0\overline{\overline{Id}}-\mu_0\left(\nabla\mathbf{u}_0+\nabla^T\mathbf{u}_0\right)\right]\cdot\mathbf{n}_i=\left[p_1\overline{\overline{Id}}-\mu_1\left(\nabla\mathbf{u}_1+\nabla^T\mathbf{u}_1\right)\right]\cdot\mathbf{n}_i+\sigma\kappa\mathbf{n}_i$$

$$\mathbf{u}_0\cdot\mathbf{n}_i=\mathbf{u}_1\cdot\mathbf{n}_i$$

$$\mathbf{u}_0\cdot\mathbf{t}_i=\mathbf{u}_1\cdot\mathbf{t}_i$$

à l'interface Σ

Vision approche discontinue de Gibbs (DNS milieux continus) :

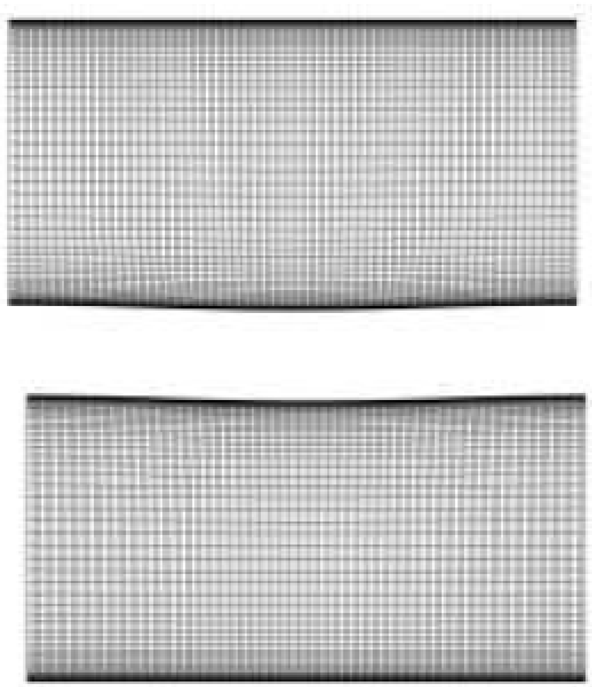
Approche lagrangienne avec suivi explicite l'interface (surface libre)

[8] S. Vincent, HDR, 2010　　*[10] Scardovelli et al., ARFM, 1986*

I. Hydrodynamique des écoulements à surface libre

I.3 Méthodes numériques

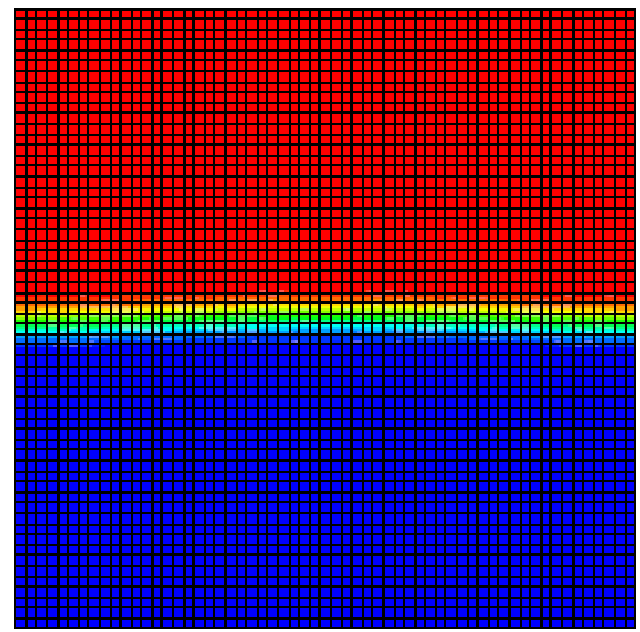
I.3.1 Vraies DNS – maillages adaptatifs



Air (milieu 0)

Surface libre Σ

Eau (milieu 1)



Approche lagrangienne DNS

Approche eulérienne MUF

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial \xi}\left(f\frac{\partial}{\partial \xi}\right)+\frac{\partial}{\partial \eta}\left(\frac{1}{f}\frac{\partial}{\partial \eta}\right)\right]\begin{pmatrix} x \\ y \end{pmatrix}=0$$

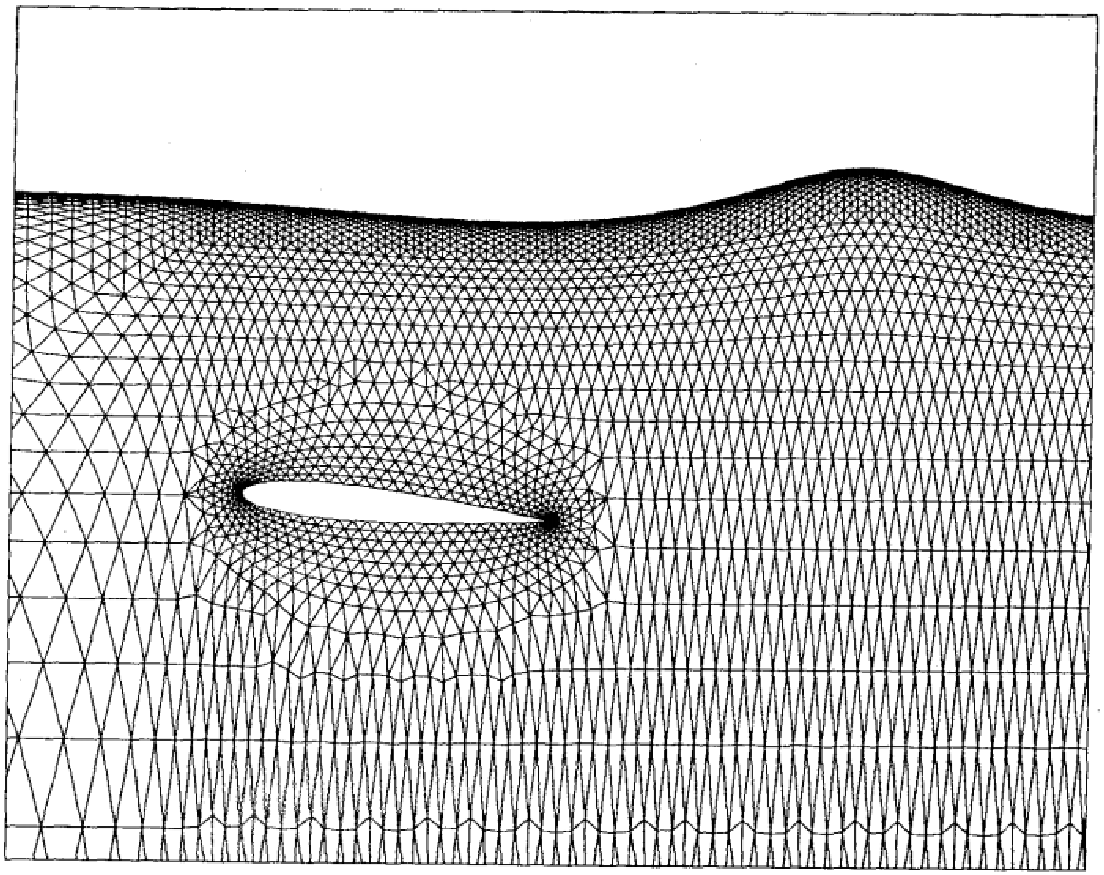
f fonction de distorsion

[11] Renaud, thèse, 2011

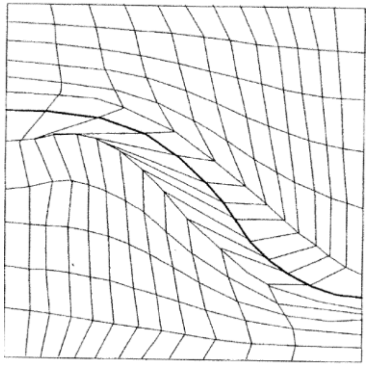
I. Hydrodynamique des écoulements à surface libre

I.3 Méthodes numériques

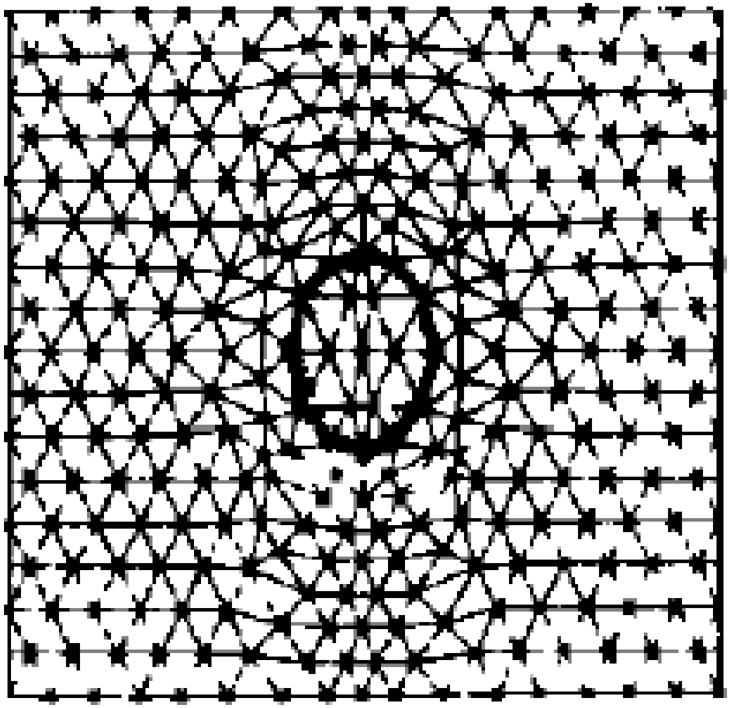
I.3.1 Vraies DNS – maillages adaptatifs



[12] Hino et al., Proc., 1993



[10] Scardovelli et al., ARFM, 1999

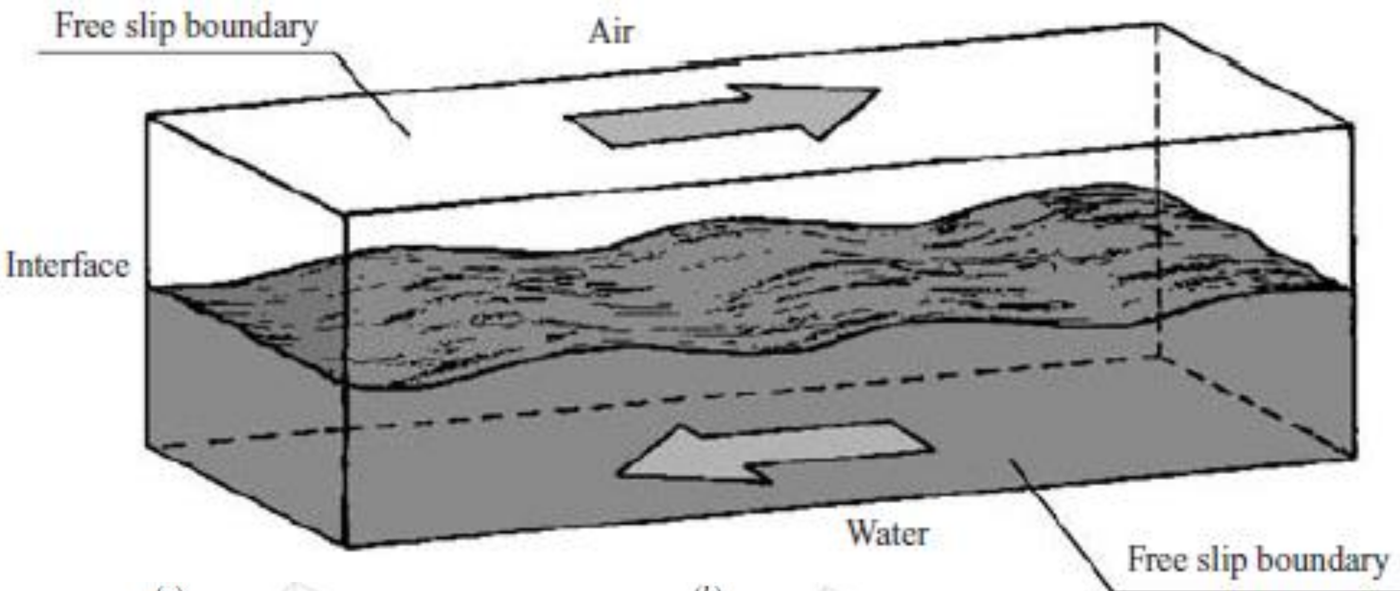


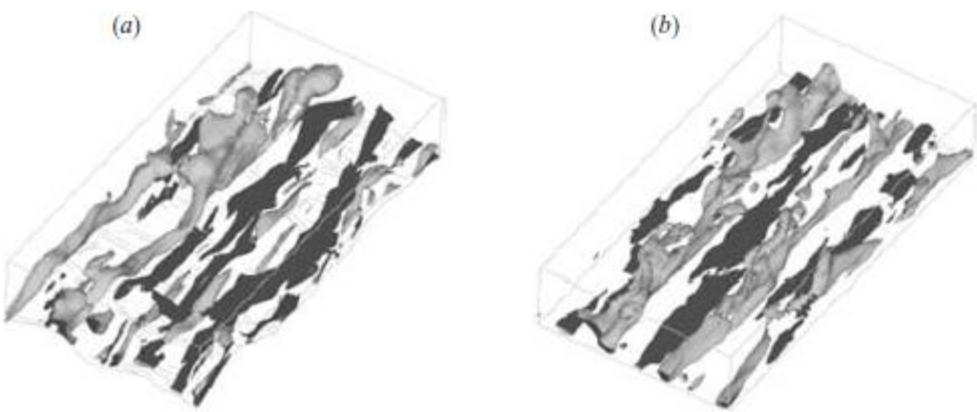
[13] Fyfe el al., JCP, 1988

I. Hydrodynamique des écoulements à surface libre

I.3 Méthodes numériques

I.3.1 Vraies DNS – maillages adaptatifs





[14] Fulgosi el al., JFM, 2003

Direct numerical simulation of turbulence in a sheared air-water flow with a ~~**deformable**~~ interface

I. Hydrodynamique des écoulements à surface libre

I.3 Méthodes numériques

I.3.2 Schémas pour les équations hyperboliques

$$\frac{\partial \mathrm{q}}{\partial t} + \frac{\partial \mathrm{F}}{\partial x} = \mathrm{S}$$

SWE

$$\frac{\partial \widetilde{C}}{\partial t} + \tilde{\mathrm{u}} \cdot \nabla \widetilde{C} = 0$$

MUF

$\Longrightarrow$ **Transport/advection d'interface, de phase, de fonction de phase, …**

Equations hyperboliques

[7] Vincent et al., JHR, 2001, MacCormack TVD superbee
[15] Trontin et al., IJNMF, 2008, WENO schemes
[16] Benkenida et al., CRAS, 1999, FCT
[17] Vincent et al., JCP, 2000, Lax-Wendroff TVD superbee

$$C_i^{n+1} = C_i^n - V\sigma(C_i^n - C_{i-1}^n) + \frac{1}{2} V\sigma\Delta x(V\sigma - 1)(\gamma_i^n - \gamma_{i-1}^n)$$

$$\gamma_i^{n,\mathrm{LW}} = C_{i+1}^n - C_i^n/\Delta x \qquad \theta_i^n = (C_i^n - C_{i-1}^n)/(C_{i+1}^n - C_i^n)$$

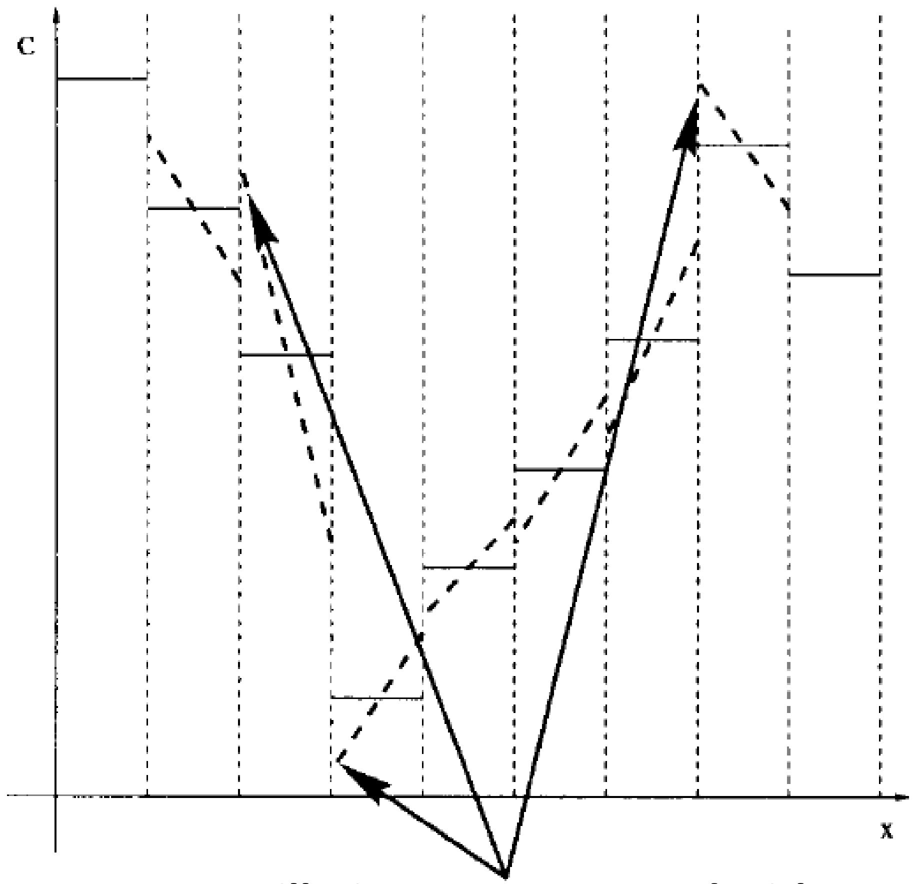
$$\gamma_i^{n,T} = \max(0, \min(1, 2\theta_i^n), \min(2, \theta_i^n)) \frac{C_{i+1}^n - C_i^n}{\Delta x} = \phi(\theta_i^n)\gamma_i^{n,LW}$$

$$\sigma = \Delta t / \Delta x$$

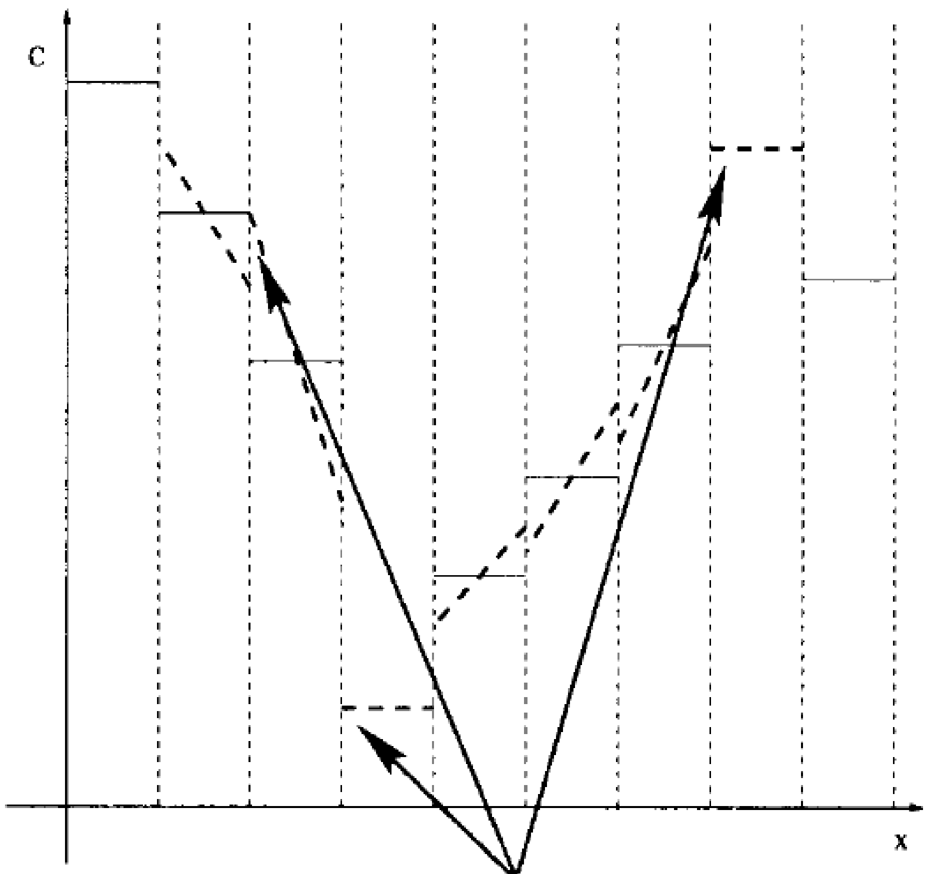
I. Hydrodynamique des écoulements à surface libre

I.3 Méthodes numériques

I.3.2 Schémas pour les équations hyperboliques



Oscillations are generated with high order schemes



Oscillations are avoided with limiters and high order schemes

I. Hydrodynamique des écoulements à surface libre

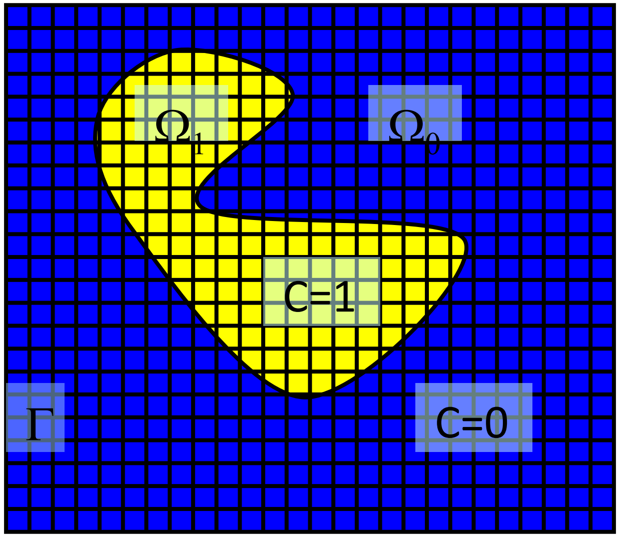
I.3 Méthodes numériques

I.3.3 Méthodes eulériennes - Suivi d'interface et frontières immergées

Level Set [15], VOF [18], Front Tracking [19]

+ AMR, solveurs, préconditionneurs

+ couplage vitesse-pression [8] différent de DNS lagrangien



Méthode time splitting [20]

$$\frac{\mathbf{u}^{n+1/2}-\mathbf{u}^n}{\Delta t}+\mathbf{u}^n\cdot\nabla\mathbf{u}^n=-\frac{1}{\rho}\nabla p^n+\mathbf{g}+\frac{1}{\rho}\nabla\cdot[\mu(\nabla\mathbf{u}^n+\nabla^T\mathbf{u}^n])+F_{TS}$$

$$\nabla\cdot\mathbf{u}^{n+1/2}=\nabla\cdot(\frac{\Delta t}{\rho}\nabla p^*)$$

$$\mathbf{u}^{n+1}=(\mathbf{u}^{n+1/2}-\frac{\Delta t}{\rho}\nabla p^*)$$

$$p^{n+1}=p^n+p^*$$

[15] Trontin et al., IJNMF, 2008 [18] Aniszewski el al., CAF, 2014 [19] Shin et al., JCP, 2002 [20] Goda, JCP, 1979

I. Hydrodynamique des écoulements à surface libre

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I.3.3 Méthodes eulériennes - Suivi d'interface et frontières immergées

Level Set [15], VOF [18], Front Tracking [19]

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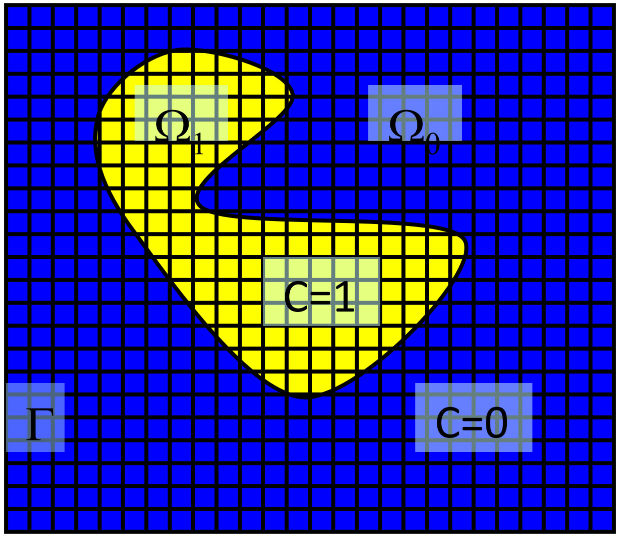

Méthode de lagrangien augmenté [21]

while $\|\nabla \cdot \mathbf{u}^{*},\mathbf{m}\| > \epsilon$, solve

$$(\mathbf{u}^{*,0}, p^{*,0}) = (\mathbf{u}^n, p^n)\rho\left(\frac{\mathbf{u}^{*,m} - \mathbf{u}^{*,0}}{\Delta t} + \mathbf{u}^{*,m-1} \cdot \nabla \mathbf{u}^{*,m}\right) - r\nabla(\nabla \cdot \mathbf{u}^{*,m})$$

$$= -\nabla p^{*,m-1} + \rho \mathbf{g} + \nabla \cdot [\mu(\nabla \mathbf{u}^{*,m} + \nabla^T \mathbf{u}^{*,m})] + \sigma k \mathbf{n}_i \delta_i p^{*,m} = p^{*,m-1} - r\nabla \cdot \mathbf{u}^{*,m}$$

[15] Trontin et al., IJNMF, 2008 [18] Aniszewski el al., CAF, 2014 [19] Shin et al., JCP, 2002 [21] Vincent et al., JCP, 2011

I. Hydrodynamique des écoulements à surface libre

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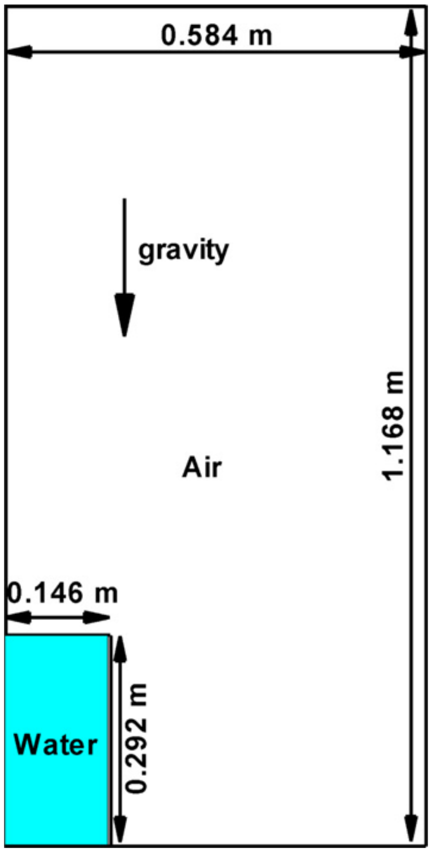
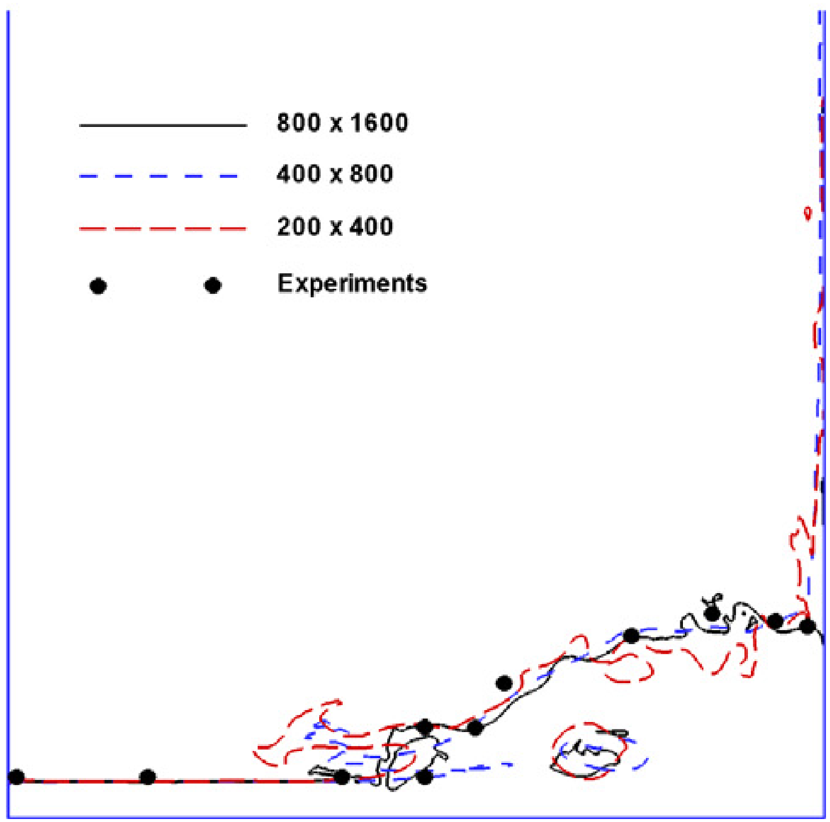
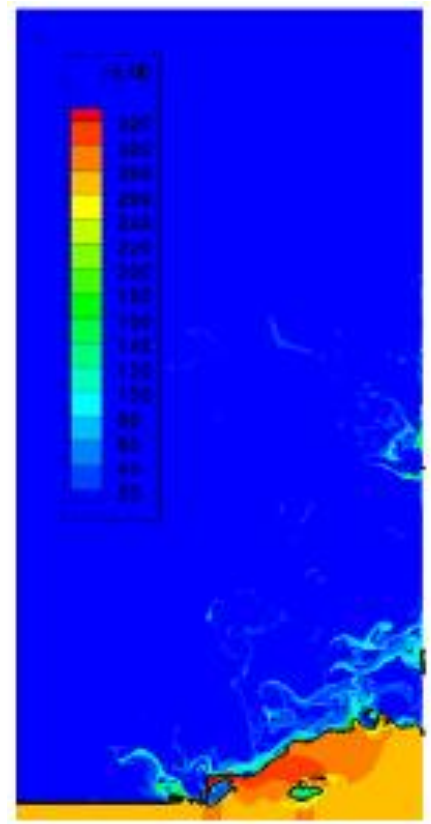
Méthode de lagrangien augmenté [21]







[21] Vincent et al., JCP, 2011

I. Hydrodynamique des écoulements à surface libre

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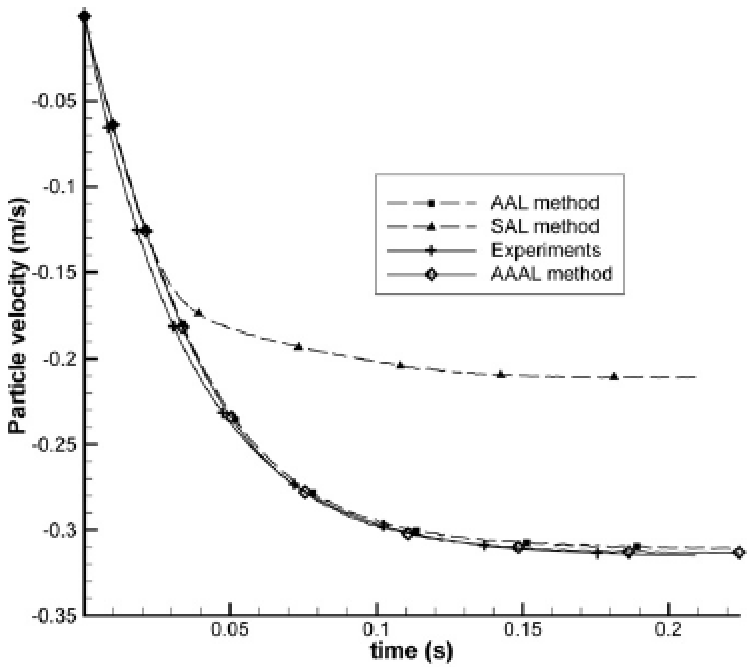
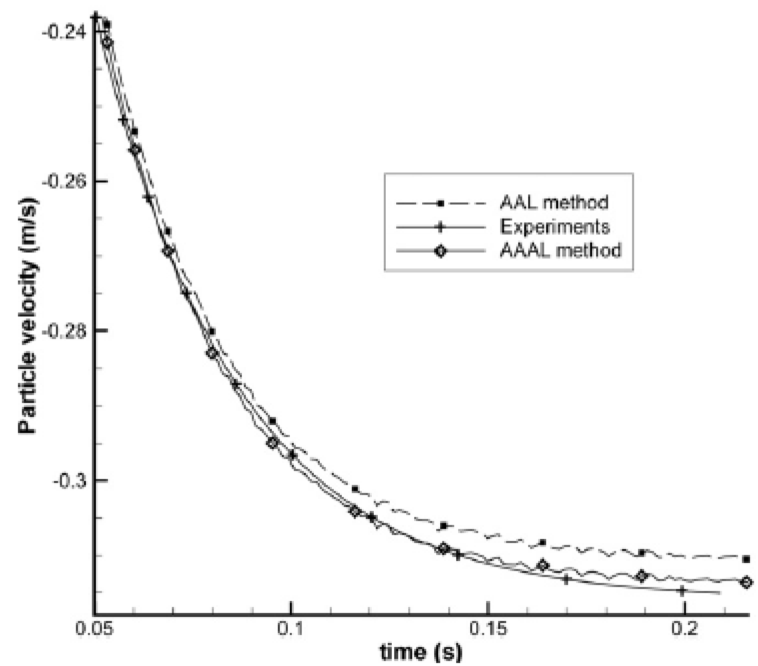
I.3.3 Méthodes eulériennes - Suivi d'interface et frontières immergées

Méthode de lagrangien augmenté [21]





$$r(t,M) = K\max\left(\rho(t,M)\frac{L_0^2}{t_0}, \rho(t,M)u_0L_0, \rho(t,M)\frac{L_0^2}{u_0}g, \frac{p_0L_0}{u_0}, \mu(t,M), \frac{\sigma}{u_0}\right)$$

[21] Vincent et al., JCP, 2011

I. Hydrodynamique des écoulements à surface libre

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Méthode de lagrangien augmenté [21]

Fig. 11. Vorticity magnitude and iso $C = 0.5$ for the free fall of a dense cylinder with 50 × 100, 100 × 200, 200 × 400, 400 × 800 and 800 × 1600 meshes.

Table 3
Relative errors and corresponding convergence orders on the fall velocities of a dense cylinder for various time steps - values obtained with the 3AL method.

Time (s)	Relative error	Order
4×10^{-4}	6.35×10^{-4}	
2×10^{-4}	2.20×10^{-4}	1.53
1×10^{-4}	1.38×10^{-4}	0.67
0.5×10^{-4}	4.34×10^{-5}	1.67
2.50×10^{-5}	3.48×10^{-5}	0.32
1.25×10^{-5}	1.0×10^{-5}	1.66
6.25×10^{-6}	2.6×10^{-6}	1.89

[21] Vincent et al., JCP, 2011

I. Hydrodynamique des écoulements à surface libre

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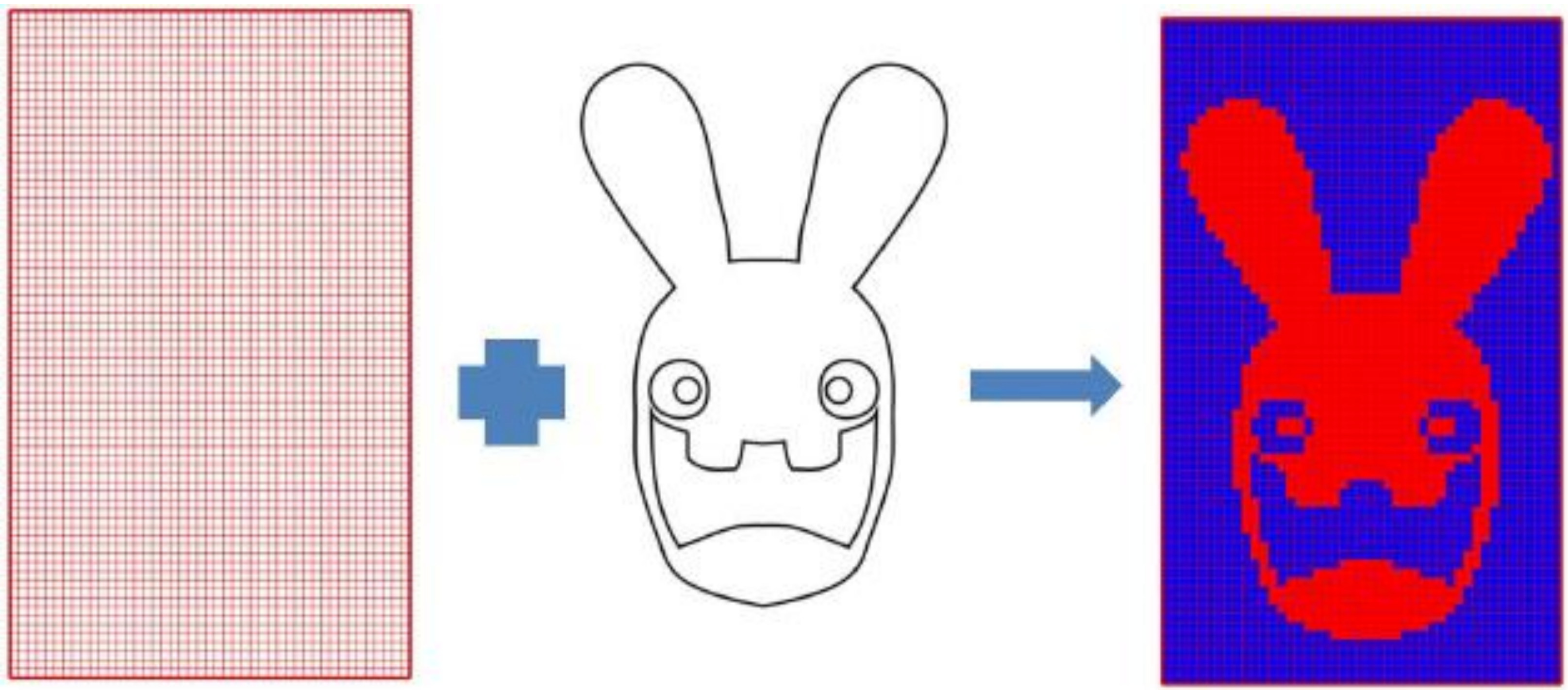
I.3.3 Méthodes eulériennes - Suivi d'interface et frontières immergées

Domaines fictifs/frontières immergées : IBM, IIM, penalisation, Cut cell, Cartesian grid, …



[8] Vincent, HDR, 2010 *[21] Vincent et al., JCP, 2011* *[24] Sarthou, Thèse, 2009*

I. Hydrodynamique des écoulements à surface libre

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I.3.3 Méthodes eulériennes - Suivi d'interface et frontières immergées

Domaines fictifs/frontières immergées : IBM, IIM, penalisation, Cut cell, Cartesian grid, …

Immersed Boundary

$$\begin{cases} -\nabla \cdot (\mathbf{a}\nabla u) = f & \text{in } \Omega_0 \\ u = u_{|\Sigma} & \text{on } \Sigma \end{cases}$$

Immersed Interface

$$\begin{cases} -\nabla \cdot (\mathbf{a}\nabla u) = f & \text{in } \Omega \\ u^-_{|\Sigma} = u_D & \text{on } \Sigma \\ u^+_{|\Sigma} = u_G & \text{on } \Sigma \end{cases}$$

$$\begin{aligned} [\![u]\!]_\Sigma = \varphi & \quad \text{on } \Sigma \\ [\![(\mathbf{a} \cdot \nabla u) \cdot \mathbf{n}]\!]_\Sigma = \psi & \quad \text{on } \Sigma \end{aligned}$$

[8] Vincent, HDR, 2010 *[21] Vincent et al., JCP, 2011* *[24] Sarthou, Thèse, 2009*

I. Hydrodynamique des écoulements à surface libre

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I.3.3 Méthodes eulériennes - Suivi d'interface et frontières immergées

Domaines fictifs/frontières immergées : IBM, IIM, penalisation, Cut cell, Cartesian grid, …

Volume penalty method

$$\begin{cases} -\nabla \cdot (a\nabla u) + Bi(u - u_D) = f \text{ in } \Omega \\ \text{with } Bi_{|\Omega_0} = 0, Bi_{|\Omega_1} = \frac{1}{\varepsilon}, \text{ for } 0 < \varepsilon \ll 1 \end{cases}$$

Immersed boundary method (IBM)

$$\rho(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\,\mathbf{u}) = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f} \text{ in } \Omega$$

$$\nabla.\mathbf{u} = 0 \text{ in } \Omega$$

$$\mathbf{f}(\mathbf{x},t) = \int_0^{L_b} \mathbf{F}(s,t)\delta(\mathbf{x} - \mathbf{X}(s,t))\, \mathrm{d}s$$

$$\frac{\partial \mathbf{X}(s,t)}{\partial t} = \mathbf{u}(\mathbf{X}(s,t),t) = \int_\Omega \mathbf{u}(\mathbf{x},t)\delta(\mathbf{x} - \mathbf{X}(s,t))\, \mathrm{d}\mathbf{x}$$

$$\mathbf{F}(s,t) = \mathbf{S}(\mathbf{X}(\cdot,t),t)$$

δ_h a discrete Dirac (or Peskin) function

[8] Vincent, HDR, 2010 *[21] Vincent et al., JCP, 2011* *[24] Sarthou, Thèse, 2009*

I. Hydrodynamique des écoulements à surface libre

I.3 Méthodes numériques

I.3.3 Méthodes eulériennes - Suivi d'interface et frontières immergées

Domaines fictifs/frontières immergées : IBM, IIM, penalisation, Cut cell, Cartesian grid, …

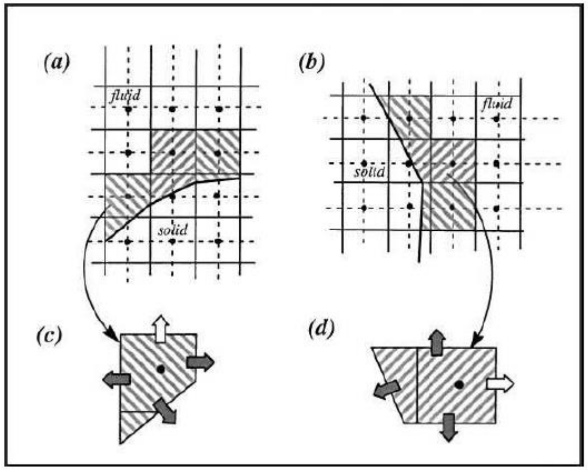
Direct forcing IBM

$$\rho\left(\frac{\mathbf{u}^{n+1}-\mathbf{u}^{n}}{\Delta t}+\left(\mathbf{u}^{n+1}.\nabla\right)\mathbf{u}^{n+1}\right)=-\nabla p^{n}+\mu\nabla^{2}\mathbf{u}^{n+1}+\delta(\mathbf{x}-\mathbf{X}(s,t))\mathbf{f}$$

Cartesian grid methods (cut cell, ghost fluid)

$$\{\nabla\cdot(a\nabla u)\}_i=\frac{1}{\Delta x}\left(a_{i+\frac{1}{2}}\left(\frac{\frac{\Delta x}{\alpha-x_i}u_{|\Gamma}-\left(\frac{x_{i+1}-\alpha}{\alpha-x_i}-1\right)u_i}{\Delta x}\right)-a_{i-\frac{1}{2}}\left(\frac{u_i-u_{i-1}}{\Delta x}\right)\right)$$

$$u^G_{i+1}=\frac{\Delta x}{\alpha-x_i}u_{|\Gamma}-\frac{x_{i+1}-\alpha}{\alpha-x_i}u_i$$



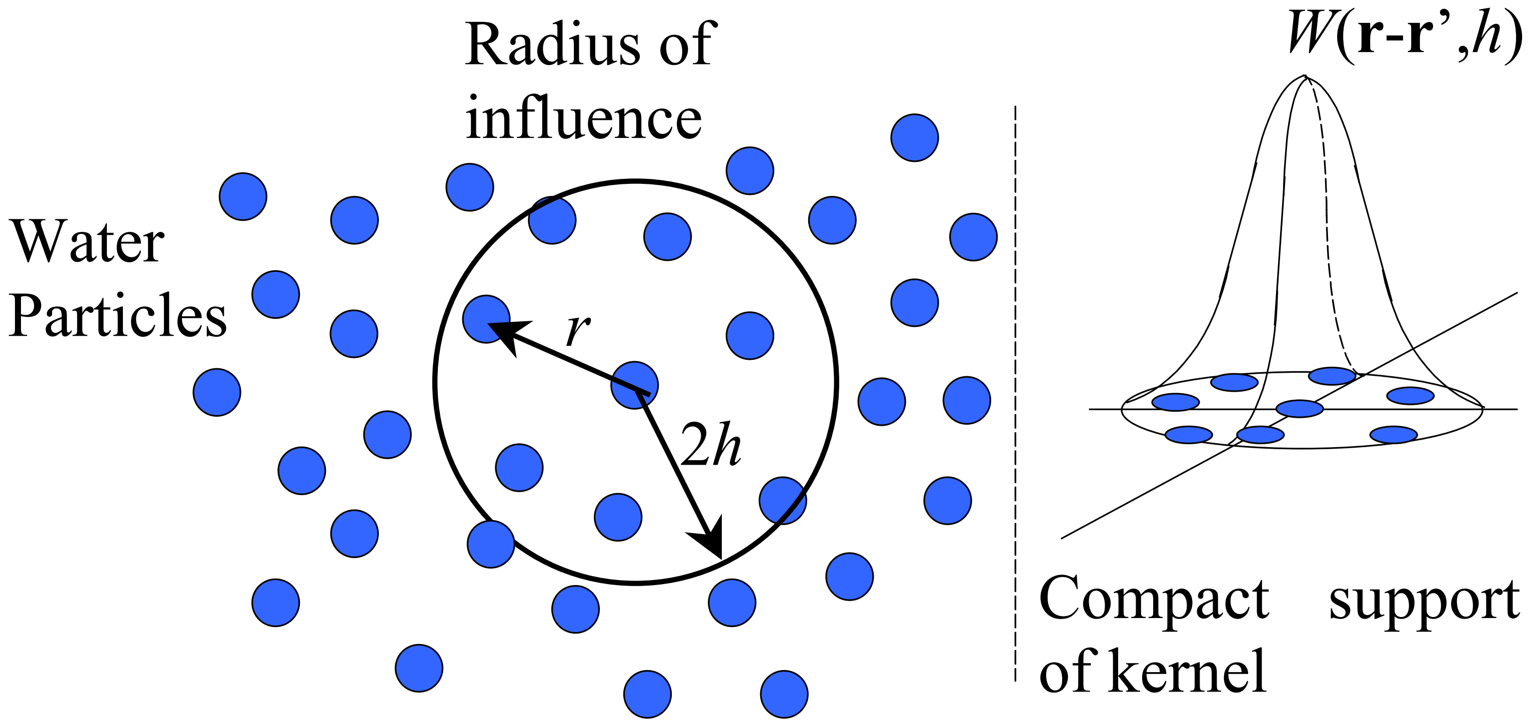
[24] Sarthou, Thèse, 2009

I. Hydrodynamique des écoulements à surface libre

I.3 Méthodes numériques

I.3.4 Méthodes lagrangiennes - SPH

The Kernel (or Weighting Function)



[22] Dalrymple et al., Coast. E, 2006

I. Hydrodynamique des écoulements à surface libre

I.3 Méthodes numériques

I.3.4 Méthodes lagrangiennes - SPH

SPH describes a fluid by replacing its continuum properties with locally (smoothed) quantities at discrete Lagrangian locations ⇒ meshless

SPH is based on integral interpolants
(Lucy 1977, Gingold & Monaghan 1977, Liu 2003)

$$A(\mathbf{r}) = \int_\Omega A(\mathbf{r}')W(\mathbf{r}-\mathbf{r}', h)\,\mathrm{d}\mathbf{r}'$$

(*W* is the smoothing kernel)

These can be approximated discretely by a summation interpolant

$$A(\mathbf{r}) \approx \sum_{j=1}^{N} A(\mathbf{r}_j)W(\mathbf{r}-\mathbf{r}_j, h)\frac{m_j}{\rho_j}$$

[22] Dalrymple et al., Coast. E, 2006

I. Hydrodynamique des écoulements à surface libre

I.3 Méthodes numériques

I.3.4 Méthodes lagrangiennes - SPH

Quadratic Kernel

$$W(r,h) = \frac{3}{2\pi h^2}\left(\frac{1}{4}q^2 - q + 1\right) \qquad q = \frac{r}{h},\ r = |\mathbf{r}_a - \mathbf{r}_b|$$

$$\nabla A_i = \sum_j \frac{m_j}{\rho_j} A_j \nabla_i W_{ij}$$

$$\rho_i (\nabla.\mathbf{u})_i = \sum_j m_j (\mathbf{u}_i - \mathbf{u}_j).\nabla_i W_{ij}$$

Advantages

Spatial gradients of the data are calculated analytically, meshless

The characteristics of the method can be changed by using a different kernel

Drawbacks

Diffuse character near sharp gradients, noisy, reseeding, shear, vorticity, turbulence

[22] Dalrymple et al., Coast. E, 2006

I. Hydrodynamique des écoulements à surface libre

I.3 Méthodes numériques

I.3.4 Méthodes lagrangiennes - SPH

Eulerian MUF

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\nabla.\mathbf{v}$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{1}{\rho}\nabla p + \upsilon_o \nabla^2 \mathbf{u} + \mathbf{F}_i$$

Lagrangian SPH

$$\frac{\mathrm{d}\mathbf{r}_i}{\mathrm{d}t} = \mathbf{v}_i + \varepsilon \sum_j m_j \left(\frac{\mathbf{v}_{ji}}{\bar{\rho}_{ij}}\right) W_{ij}$$

$$\frac{\mathrm{d}\rho_i}{\mathrm{d}t} = \sum_j m_j \left(\mathbf{v}_i - \mathbf{v}_j\right) \cdot \nabla_i W_{ij}$$

$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij}\right) \nabla_i W_{ij} + \mathbf{F}_i$$

$$\left(\frac{\mathrm{d}m_i}{\mathrm{d}t} = 0\right)$$

[22] Dalrymple et al., Coast. E, 2006

I. Hydrodynamique des écoulements à surface libre

I.3 Méthodes numériques

I.3.4 Méthodes lagrangiennes - SPH

Lagrangian SPH

$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij} \right) \nabla_i W_{ij} + \mathbf{F}_i$$

Equation of state (Batchelor 1974):

$$p = B\left(\left[\frac{\rho}{\rho_o} \right]^{\gamma} - 1 \right)$$

accounts for incompressible flows by setting B such that speed of sound is

$$c = \sqrt{\frac{\mathrm{d}p}{\mathrm{d}\rho}} \approx 10 v_{\max}$$

Compressibility
$O(M^2)$

Viscosity generally accounted for by an artificial empirical term (Monaghan 1992):

$$\Pi_{ij} = \begin{cases} \dfrac{-\alpha \bar{c}_{ij} \mu_{ij}}{\bar{\rho}_{ij}} & \mathbf{v}_{ij}.\mathbf{r}_{ij} < 0 \\ 0 & \mathbf{v}_{ij}.\mathbf{r}_{ij} \geq 0 \end{cases}$$

$$\mu_{ij} = \frac{h \mathbf{v}_{ij}.\mathbf{r}_{ij}}{r_{ij}^2 + \eta^2}$$

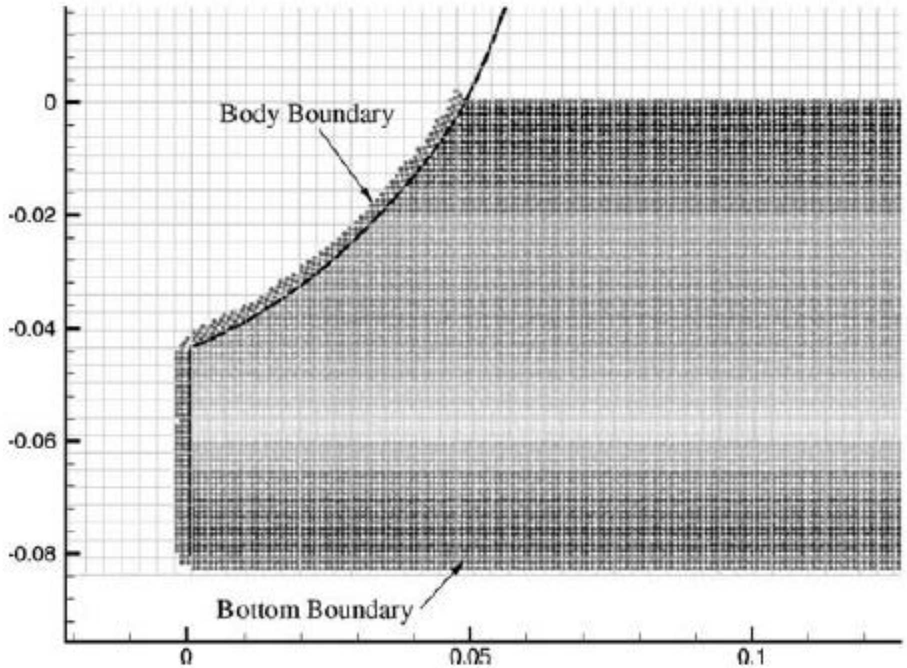
[22] Dalrymple et al., Coast. E, 2006

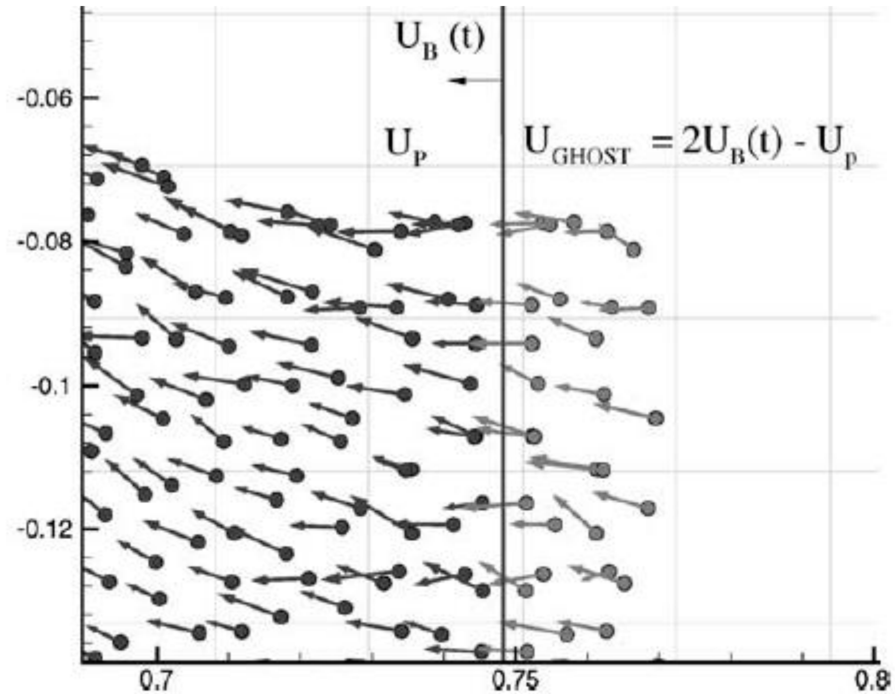
I. Hydrodynamique des écoulements à surface libre

I.3 Méthodes numériques

I.3.4 Méthodes lagrangiennes - SPH

Lagrangian SPH (boundary conditions)





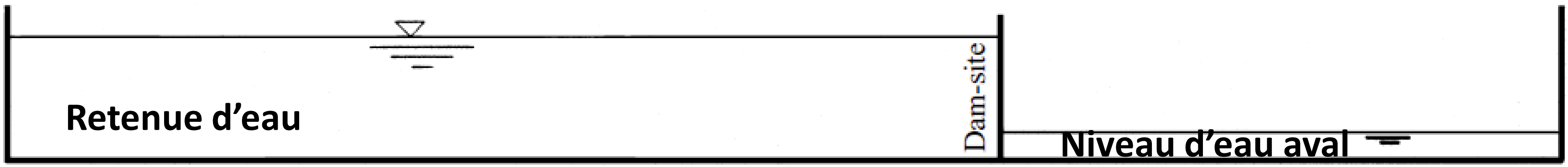
Effet mirroir / particules fantomes (bounce back)

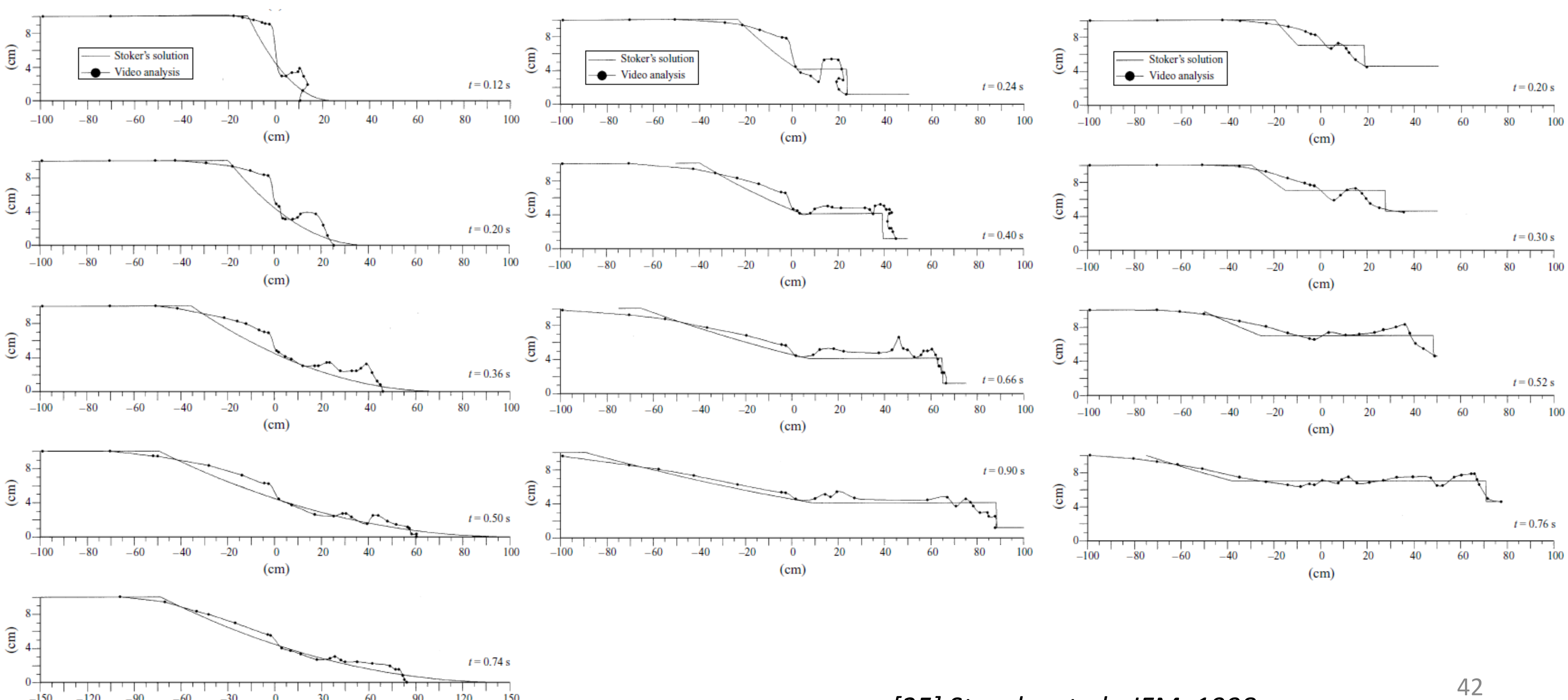
$$x_{iG} = 2x_{\mathrm{w}} - x_i, \quad u_{niG} = 2U_{\mathrm{nw}} - u_{ni}, \quad p_{iG} = p_i$$
$$u_{tiG} = u_{ti},$$

[23] Calagrossi et al., JCP, 2003

II. Exemple de la rupture de barrage

II.1 Résultats de référence

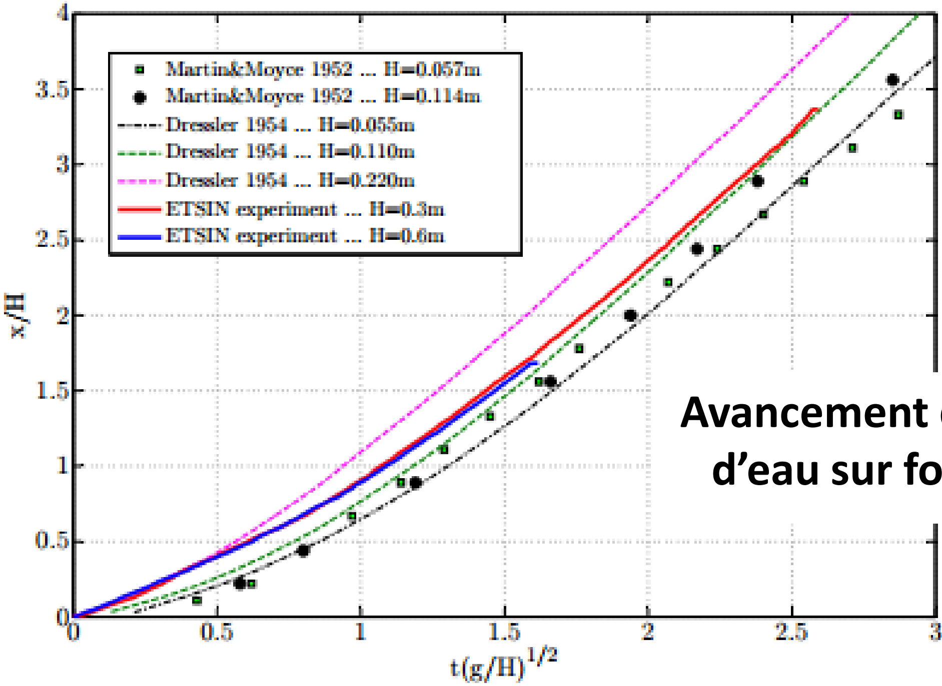


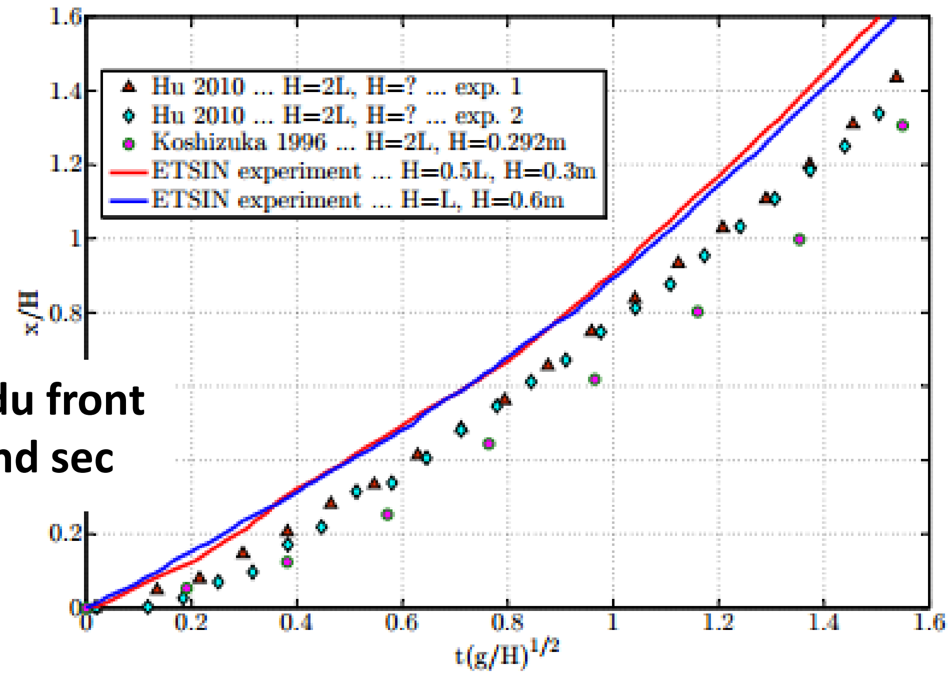


[25] Stansby et al., JFM, 1998

II. Exemple de la rupture de barrage

II.1 Résultats de référence





Avancement du front d'eau sur fond sec

$$\begin{cases} f_a(t) = 2\dfrac{x_a(t)}{x_a(0)} \\ h_{s,a}(t) = \dfrac{h_a(t)}{h_g} \\ t_a = 2t\sqrt{\dfrac{g}{x_a(0)}} \end{cases}$$

$$I_v = ||\nabla \wedge \mathbf{u}||_2 = \left| \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \right|$$

$$Re = \frac{\rho(h_g - h_d)\sqrt{g h_g}}{\mu}$$

[26] Lobovsky et al., JFS, 2013

[6] Vincent, Thèse, 1999

II. Exemple de la rupture de barrage

II.1 Résultats de référence

Rupture de barrage sur fond sec

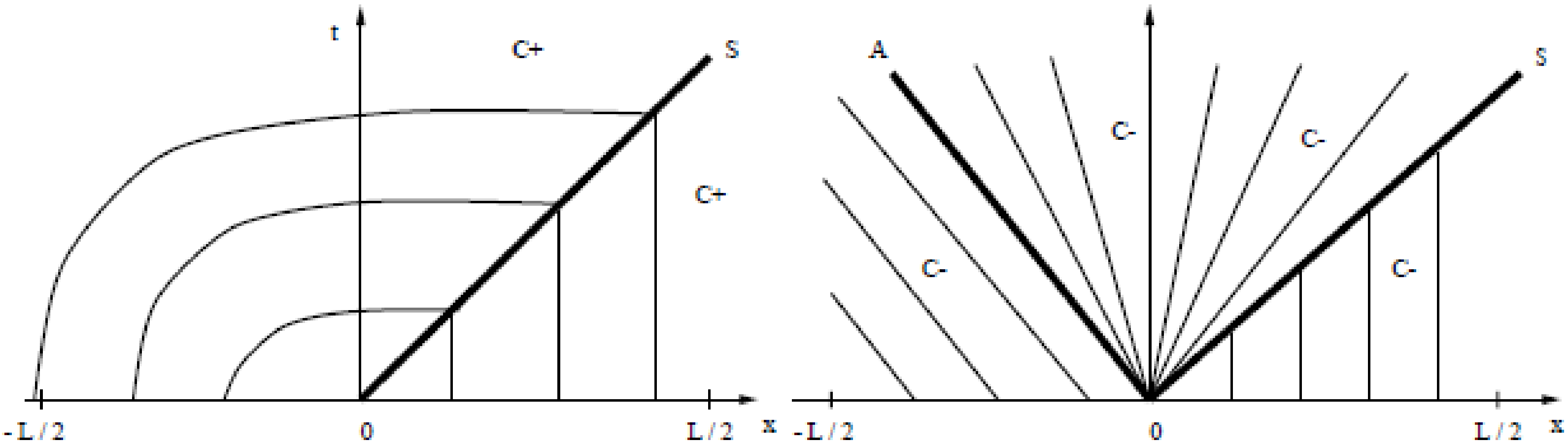


Figure 3.22 : *Left: description of the $C_- = u - \sqrt{gh}$ characteristics curves in the (x,t) plan - Initially, the dam-break is located in x=0 - an expansion wave appears between A and $x = 0$. Right: description of the $C_+ = u + \sqrt{gh}$ characteristics curves in the (x,t) plan - a shock wave appears in S*

[7] Vincent et al., JHR, 2001

II. Exemple de la rupture de barrage

II.1 Résultats de référence

Rupture de barrage sur fond sec

The analytical solution is separated into three zones:

- from x = -L/2 to OA, we obtain a steady state corresponding to initial conditions. OA is the $x = -\sqrt{gh_1}t$ curve. The analytical solution is u = 0 and $h = h_1$

- from OA to OS, we get an expansion wave containing the sonic point x = 0. OS is the $x = 2\sqrt{gh_1}t$ curve. The solution is $u = (\frac{2}{3}\sqrt{gh_1} + \frac{2}{3}\frac{x}{t})$ and $h = \dfrac{(\frac{2}{3}\sqrt{gh_1} - \frac{x}{3t})^2}{g}$

- from OS to x = L/2, we obtain a steady state corresponding to initial conditions. The solution is u = 0 and $h = h_2$

[7] Vincent et al., JHR, 2001

II. Exemple de la rupture de barrage

II.1 Résultats de référence

Rupture de barrage sur fond sec

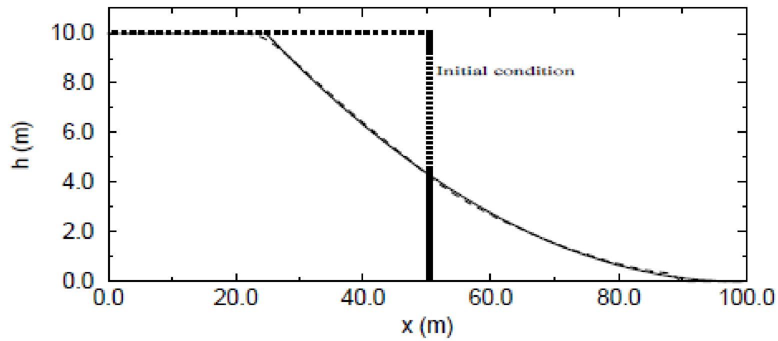


Figure 3.23 : *Water surface profile at $t = 2.5$ s - Comparison between the analytical solution (——) and the simulation (- - -) with the McCormack TVD scheme. 200 grid points are computed with $\Delta t = 0.01$.*

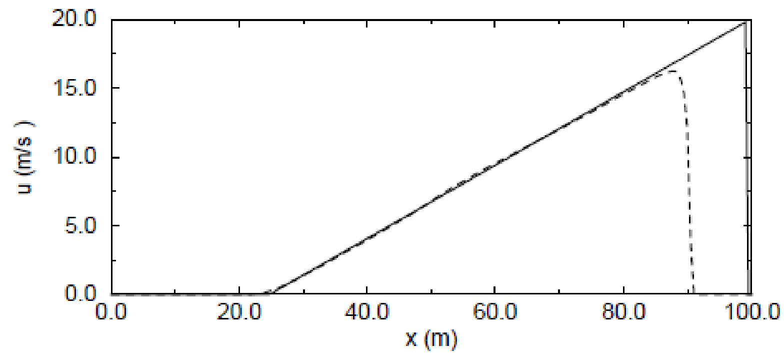


Figure 3.24 : *Velocity profile at $t = 2.5$ s - Comparison between the analytical solution (——) and the simulation (- - -) with the McCormack TVD scheme. 200 grid points are computed with $\Delta t = 0.01$.*

[7] Vincent et al., JHR, 2001

II. Exemple de la rupture de barrage

II.1 Résultats de référence

Rupture de barrage sur fond mouillé

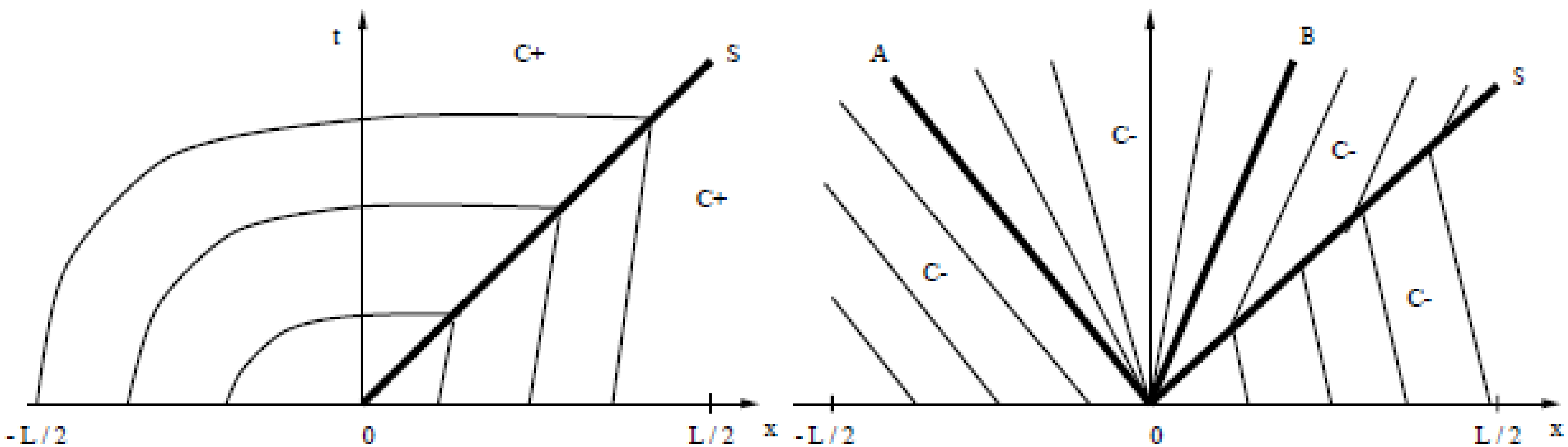


Figure 3.19 : *Left: description of the* $C_+ = u + \sqrt{gh}$ *characteristics curves in the* (x, t) *plan. Initially, the dam–break is located in x=0. A shock wave S appears. Right: description of the* $C_- = u - \sqrt{gh}$ *characteristics curves in the* (x, t) *plan. An expansion wave appears between A and B.*

[7] Vincent et al., JHR, 2001

II. Exemple de la rupture de barrage

II.1 Résultats de référence

Rupture de barrage sur fond mouillé

Leading the characteristic theory, the analytical solution is then:

- from x = -L/2 to OA (OA is the $x = -\sqrt{gh_1}t$ curve), the analytical solution is u = 0 and $h = h_1$.

- from OA to OB (OB is the $x = h_s t$ curve), we get $u = (\frac{2}{3}\sqrt{gh_1} + \frac{2}{3}\frac{x}{t})$ and $h = \dfrac{(\frac{2}{3}\sqrt{gh_1} - \frac{x}{3t})^2}{g}$.

- from OB to OS (OS corresponds to the x = s t curve, where s is the shock velocity), the solutions h_s and u_s are obtained solving the following system

$$\begin{cases} u_s + 2\sqrt{gh_s} - 2\sqrt{gh_1} = 0 \text{ (Riemann invariant)} \\ s(h_2 - h_s) - (h_2u_2 - h_su_s) = 0 \text{ (flux continuity through the shock)} \\ s(h_2u_2 - h_su_s) - (\frac{(h_2u_2)^2}{h_2} + \frac{gh_2^2}{2}) + (\frac{(h_su_s)^2}{h_s} + \frac{gh_s^2}{2}) = 0 \text{ (flux continuity through the shock)} \end{cases}$$

- from OS to x = L/2, the solution is u = 0 and $h = h_2$

[7] Vincent et al., JHR, 2001

II. Exemple de la rupture de barrage

II.1 Résultats de référence

Rupture de barrage sur fond mouillé

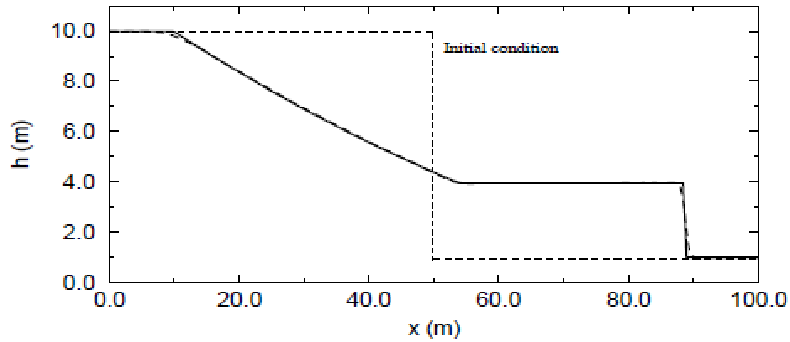


Figure 3.20 : *Water surface profile at $t = 4$ s - Comparison between the analytical solution (——) and the simulation (- - -) with the McCormack TVD scheme. 200 grid points are computed with $\Delta t = 0.01$.*

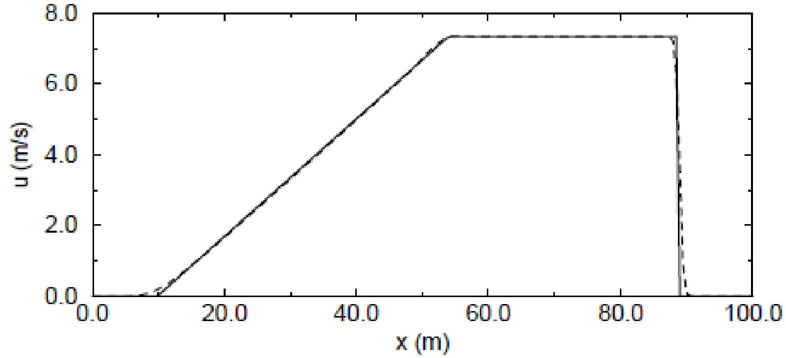


Figure 3.21 : *Velocity profile at $t = 4$ s - Comparison between the analytical solution (——) and the simulation (- - -) with the McCormack TVD scheme. 200 grid points are computed with $\Delta t = 0.01$.*

[7] Vincent et al., JHR, 2001

II. Exemple de la rupture de barrage

II.2 Comparaison entre théorie, approches intégrées et simulation directe

Nom du problème	$h_g(m)$	$h_d(m)$	L_b
RB1	0.1	0.	0.6
RB2	0.1	0.01	0.6
RB3	0.1	0.045	0.6
RB4	0.1	0.	0.1

$$\begin{cases} \rho = 1000 kg.m^{-3} \text{ et } \mu = 10^{-3} kg.m^{-1}.s^{-1} \text{ dans l'eau} \\ \rho = 1.1768 kg.m^{-3} \text{ et } \mu = 1.85 \cdot 10^{-5} kg.m^{-1}.s^{-1} \text{ dans l'air} \end{cases}$$

$$maillage\ 600\ x\ 70,\ \Delta x = 0.002m,\ \Delta t = 0.0003s$$

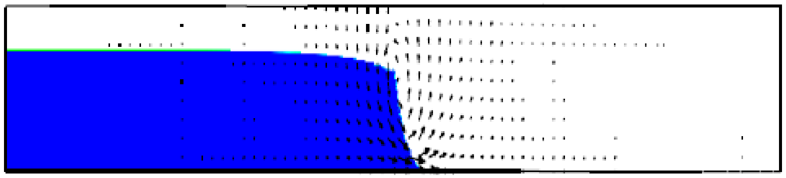
[6] Vincent, Thèse, 1999

II. Exemple de la rupture de barrage

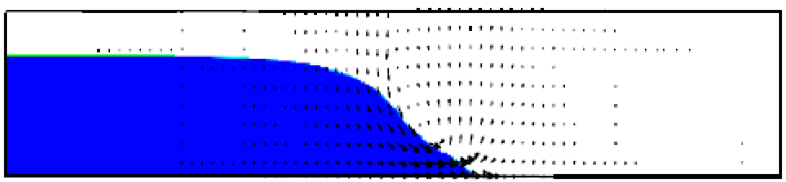
II.2 Comparaison entre théorie, approches intégrées et simulation directe

Simulations VOF

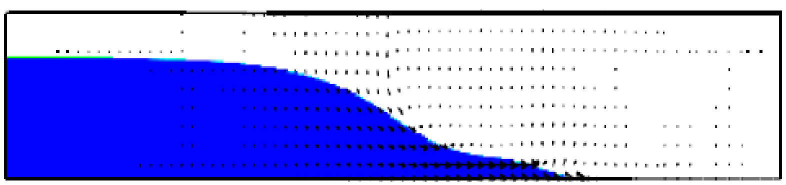
RB1



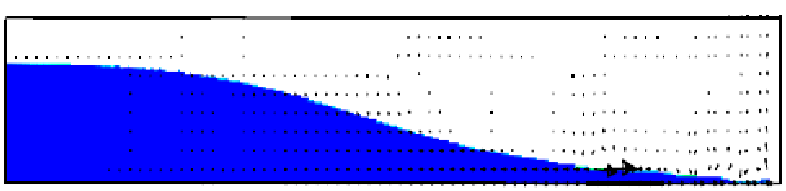
(a) $t = 0.06s$



(b) $t = 0.12s$

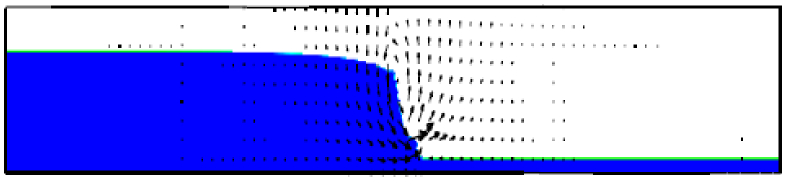


(c) $t = 0.2s$

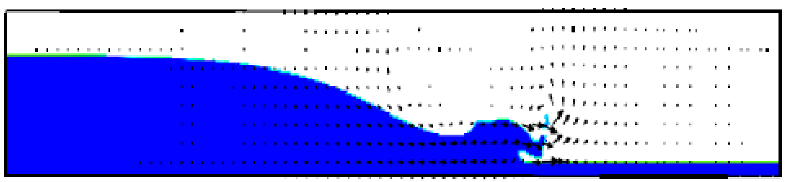


(d) $t = 0.36s$

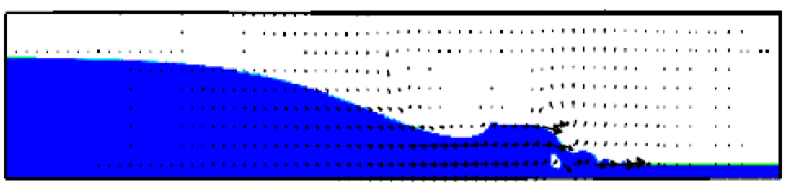
RB2



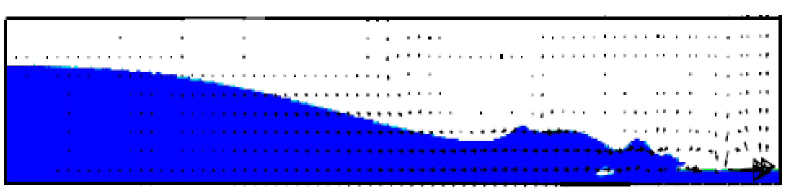
(a) $t = 0.06s$



(b) $t = 0.24s$



(c) $t = 0.3s$



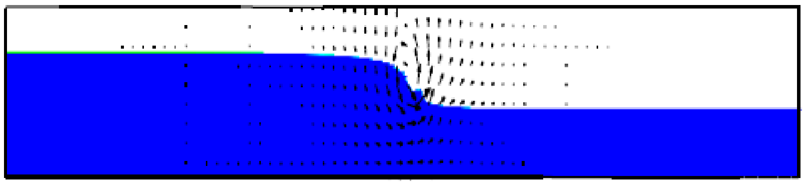
(d) $t = 0.42s$

[6] Vincent, Thèse, 1999

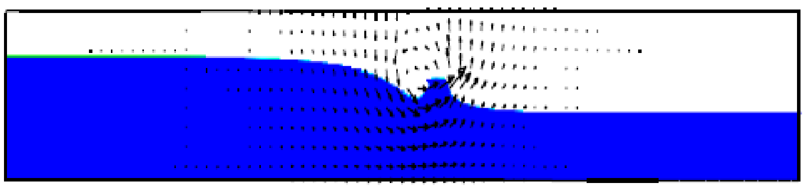
II. Exemple de la rupture de barrage

II.2 Comparaison entre théorie, approches intégrées et simulation directe

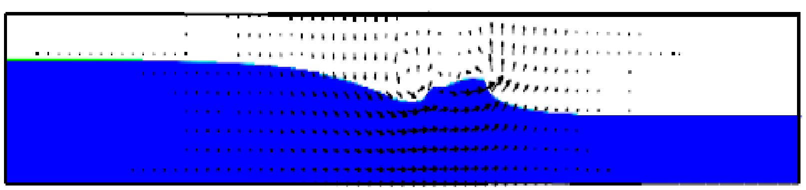
RB3



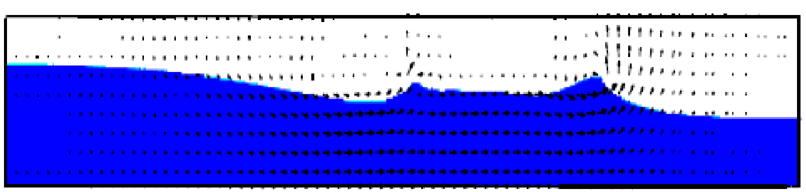
(a) $t = 0.06s$



(b) $t = 0.12s$



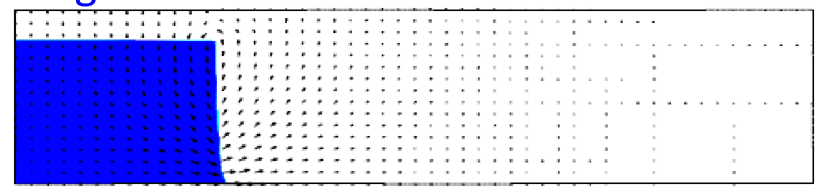
(c) $t = 0.2s$



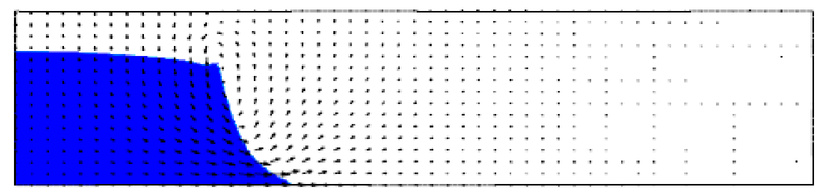
(d) $t = 0.42s$

Simulations VOF

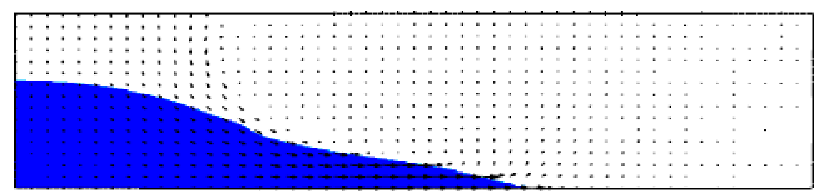
RB4



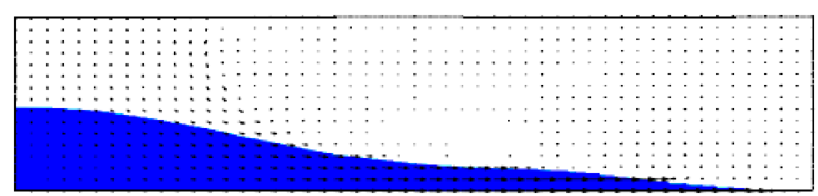
(a) $t = 0.02s$



(b) $t = 0.06s$



(c) $t = 0.14s$

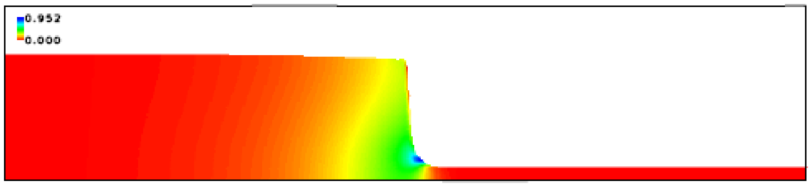


(d) $t = 0.2s$

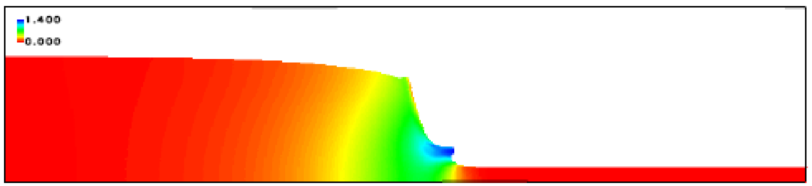
[6] Vincent, Thèse, 1999

II. Exemple de la rupture de barrage

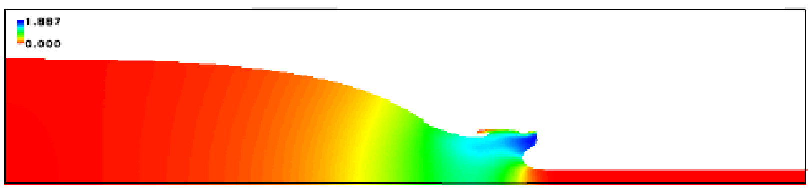
II.2 Comparaison entre théorie, approches intégrées et simulation directe



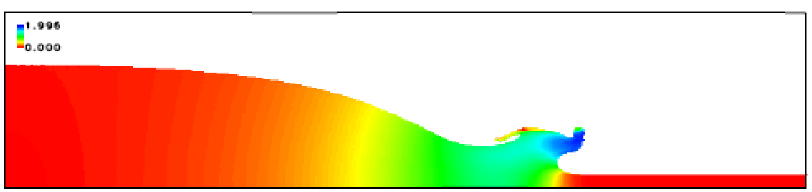
(a) $t = 0.03s$



(b) $t = 0.06s$



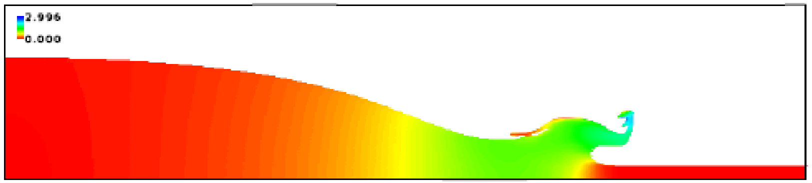
(c) $t = 0.12s$



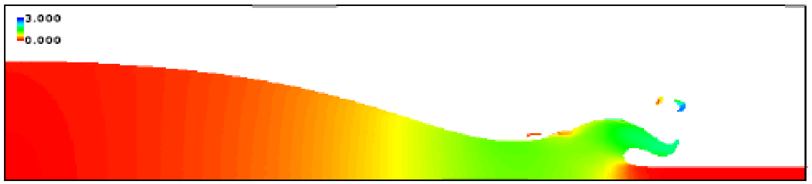
(d) $t = 0.15s$

Simulations VOF

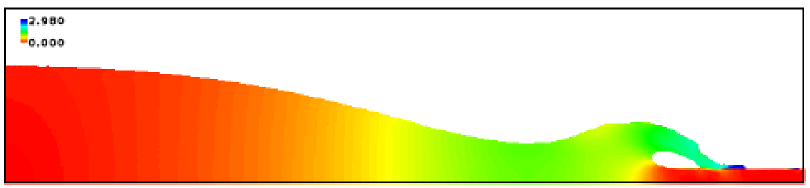
Froude RB2



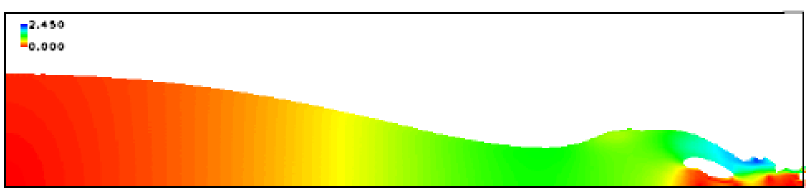
(a) $t = 0.18s$



(b) $t = 0.21s$



(c) $t = 0.24s$



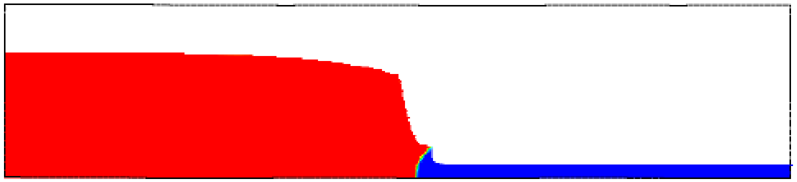
(d) $t = 0.27s$

[6] Vincent, Thèse, 1999

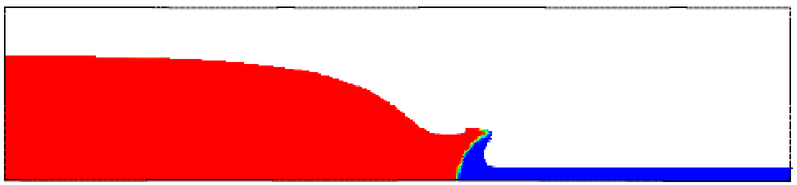
II. Exemple de la rupture de barrage

II.2 Comparaison entre théorie, approches intégrées et simulation directe

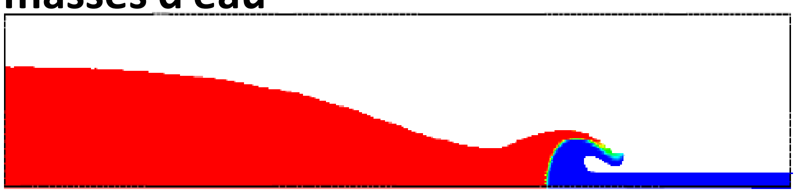
Distribution des masses d'eau



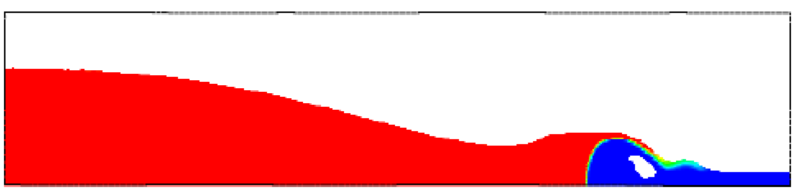
(a) $t = 0.06s$



(b) $t = 0.12s$



(c) $t = 0.24s$

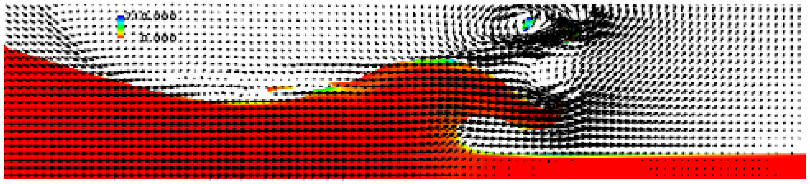


(d) $t = 0.3s$

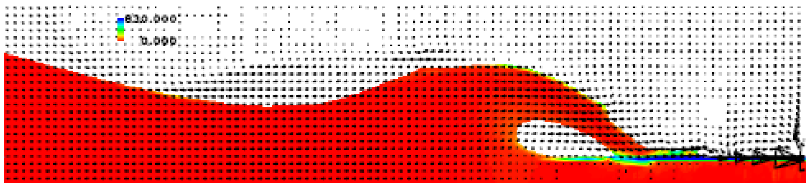
Simulations VOF

RB2

Sans capillarité

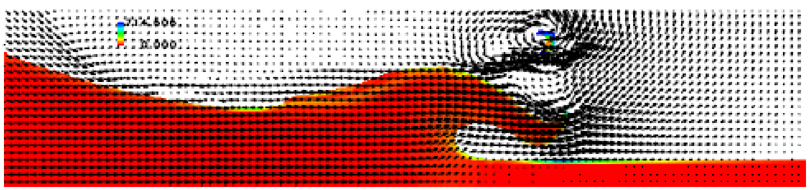


(a) $t = 0.21s$, $We = +\infty$

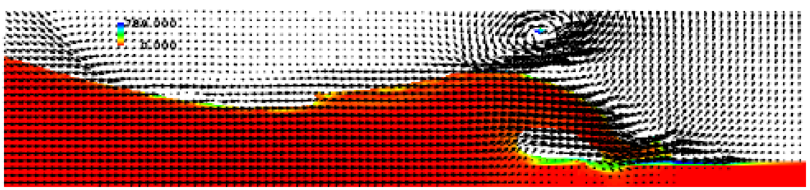


(b) $t = 0.24s$, $We = +\infty$

Avec capillarité



(c) $t = 0.21s$, $We = 1176$

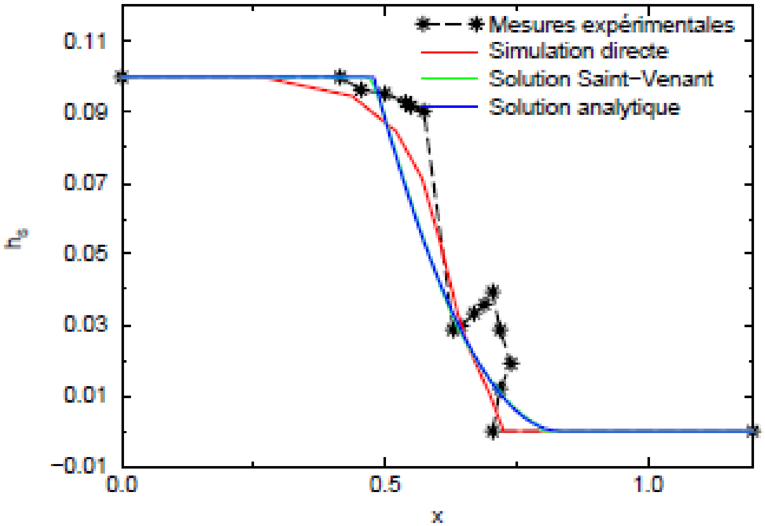


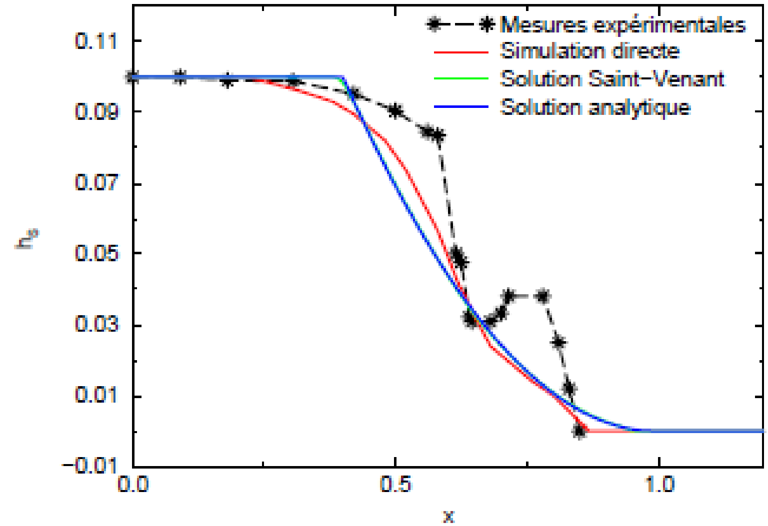
(d) $t = 0.24s$, $We = 1176$

[6] Vincent, Thèse, 1999

II. Exemple de la rupture de barrage

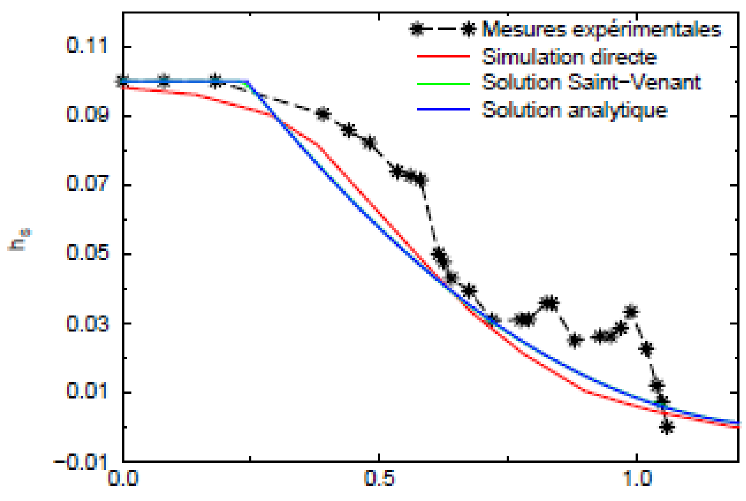
II.2 Comparaison entre théorie, approches intégrées et simulation directe





Cas RB1 :

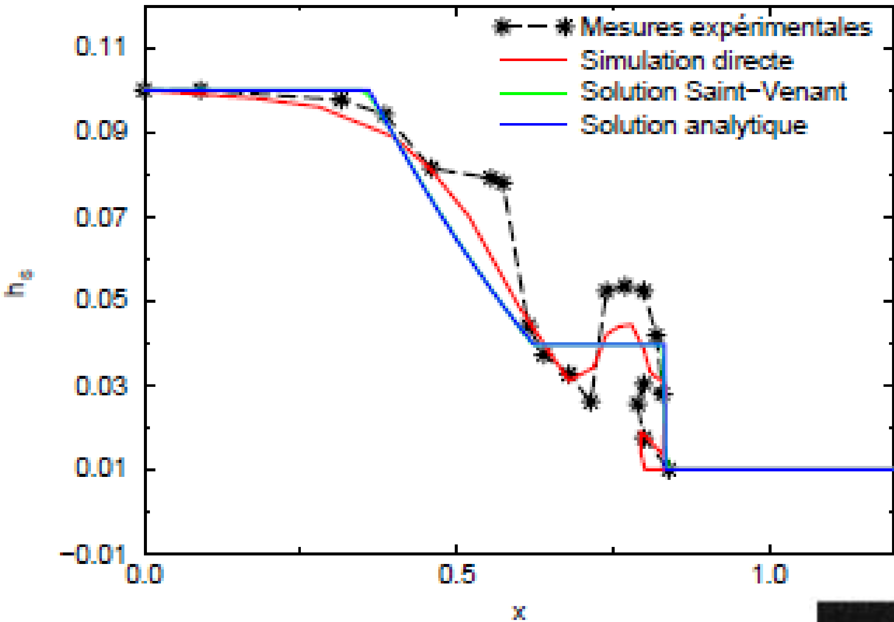
Comparaison exp, VOF et SWE

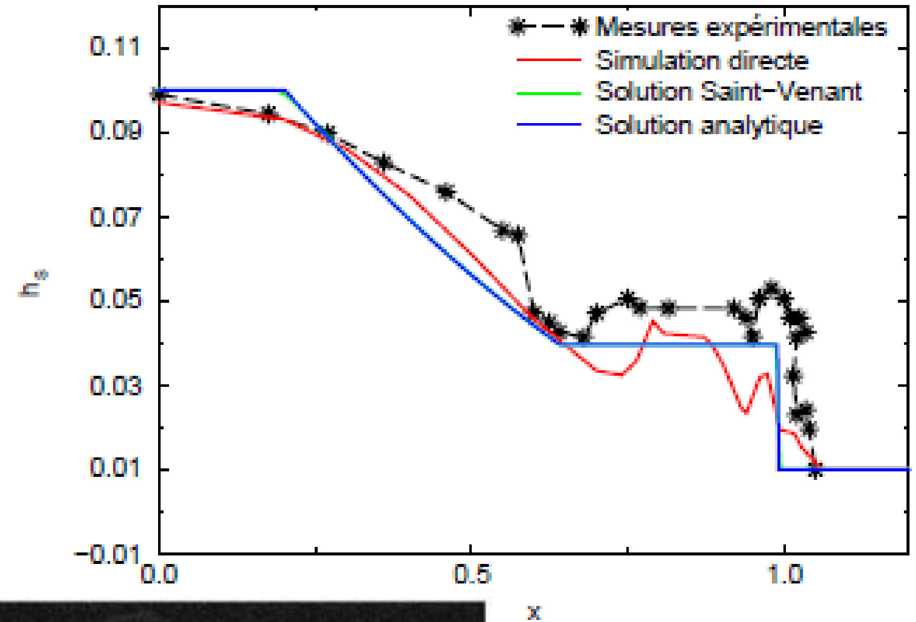


[6] Vincent, Thèse, 1999

II. Exemple de la rupture de barrage

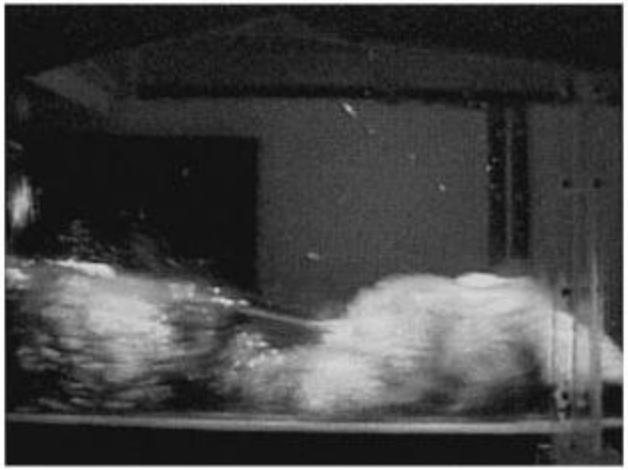
II.2 Comparaison entre théorie, approches intégrées et simulation directe





Cas RB2 :

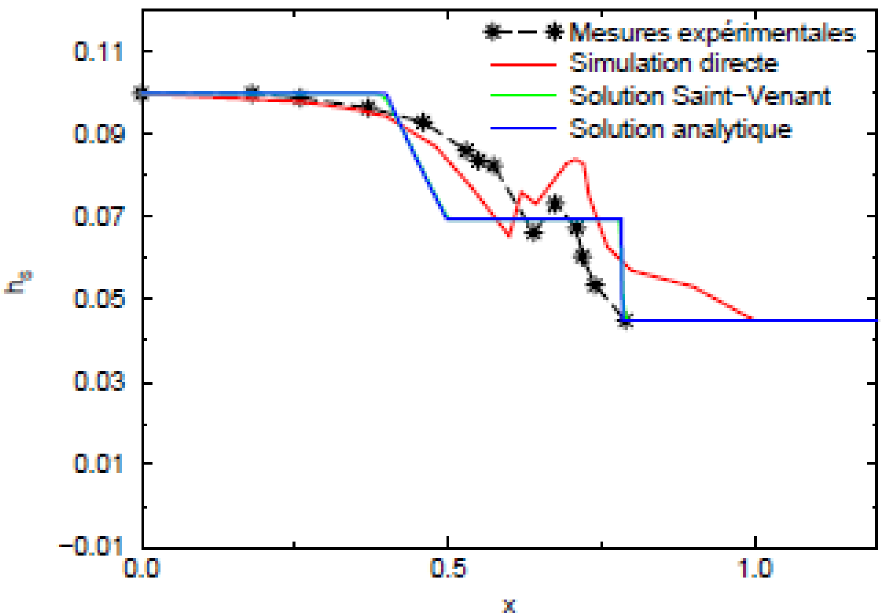
Comparaison exp, VOF et SWE

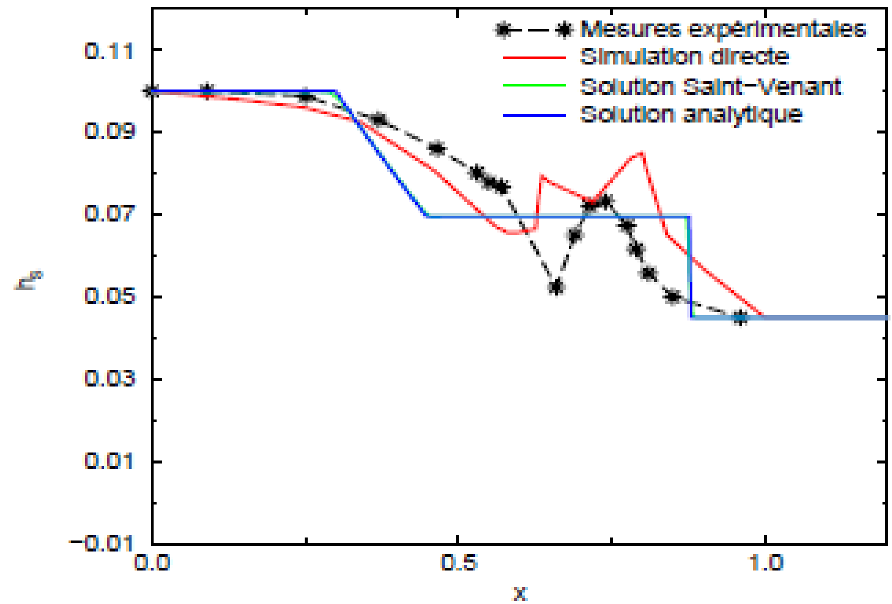


[6] Vincent, Thèse, 1999

II. Exemple de la rupture de barrage

II.2 Comparaison entre théorie, approches intégrées et simulation directe





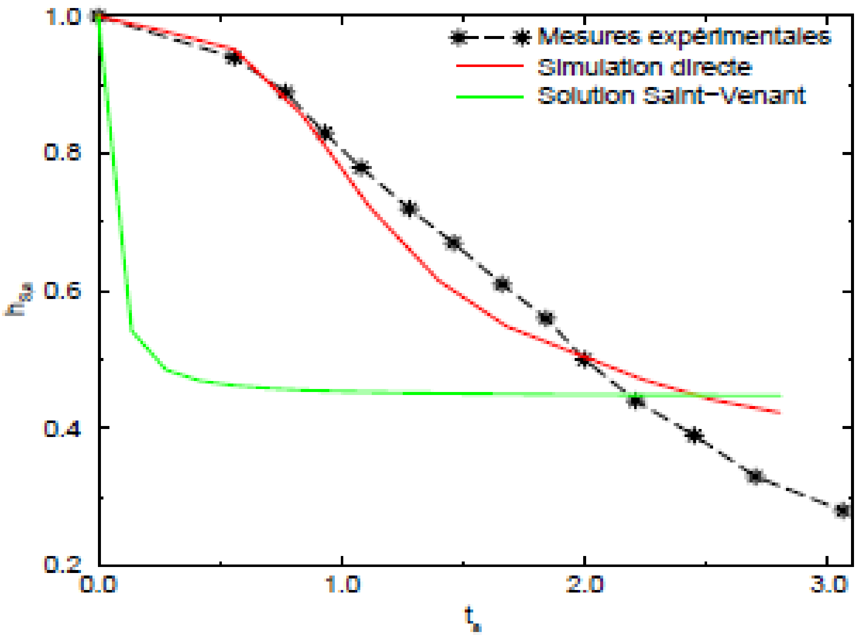
Cas RB3 :

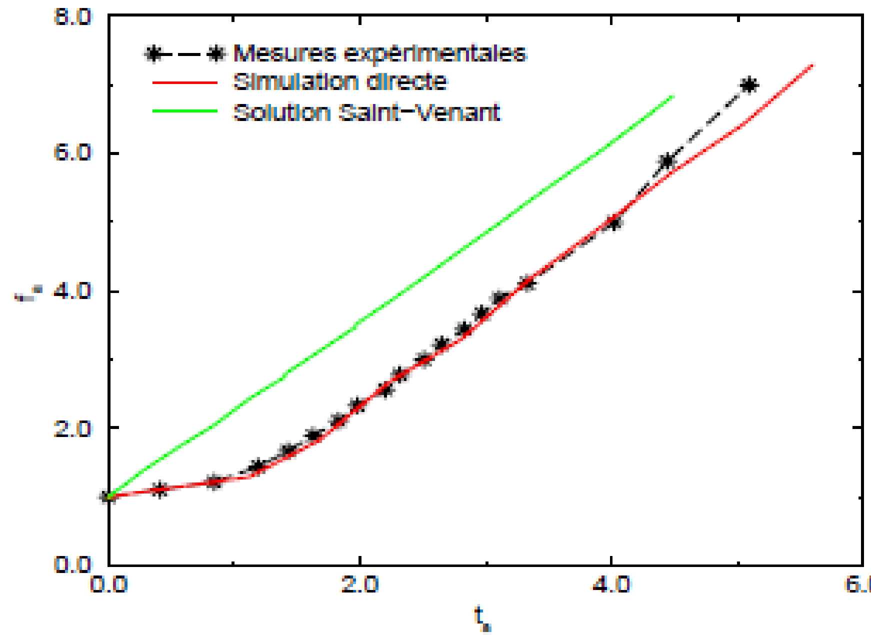
Comparaison exp, VOF et SWE

[6] Vincent, Thèse, 1999

II. Exemple de la rupture de barrage

II.2 Comparaison entre théorie, approches intégrées et simulation directe





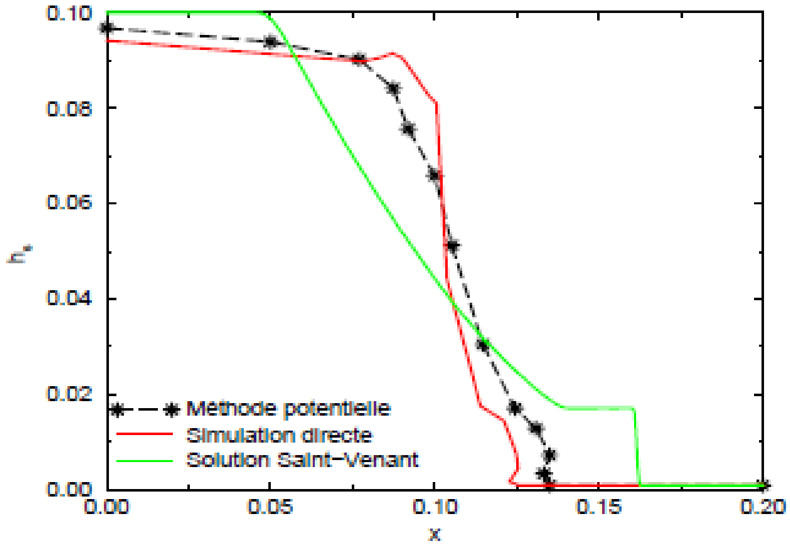
Cas RB1 :

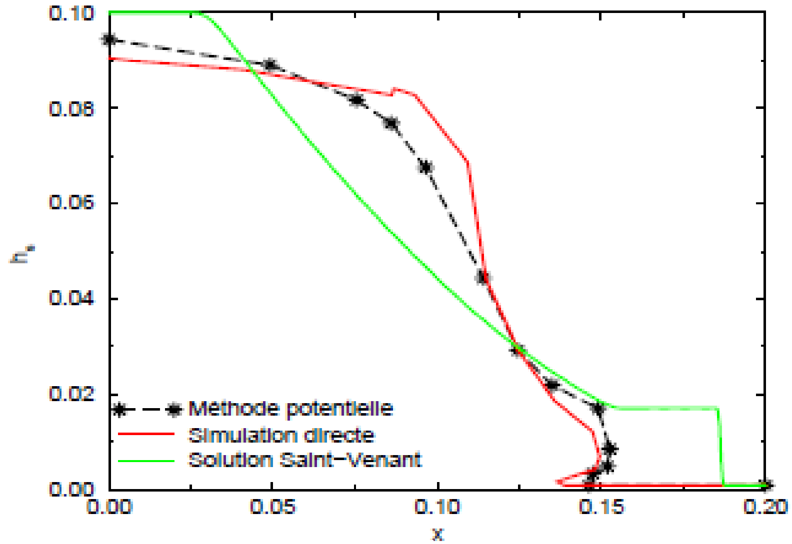
Comparaison exp, VOF et SWE

[6] Vincent, Thèse, 1999

II. Exemple de la rupture de barrage

II.2 Comparaison entre théorie, approches intégrées et simulation directe



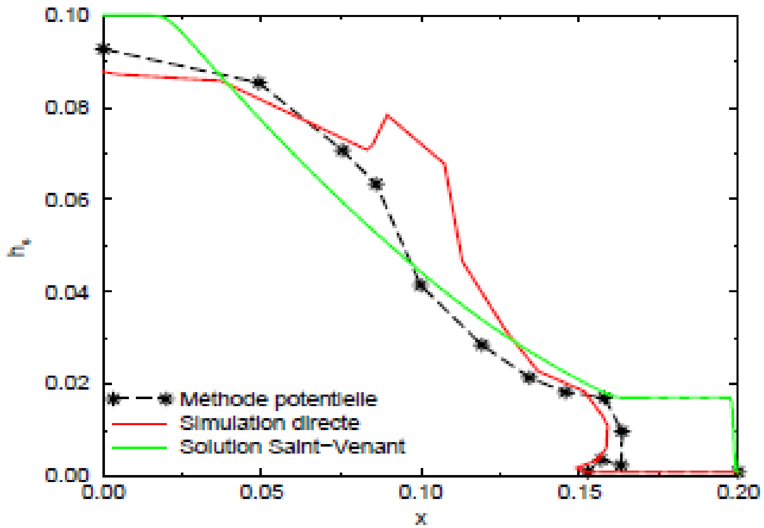


Cas RB1 :

Comparaison exp, VOF et SWE

Avec fond mouillé

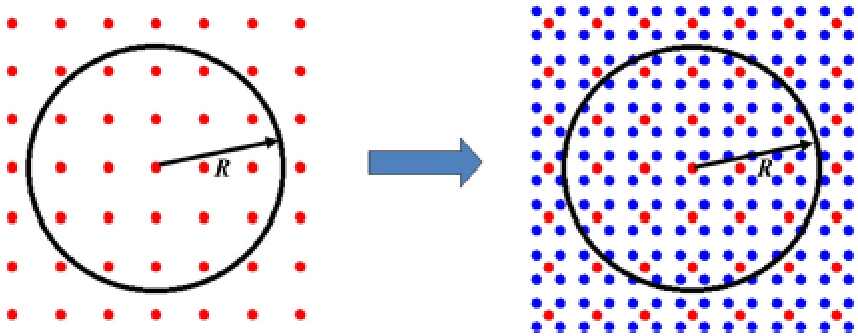
h_d=0.001 m



[6] Vincent, Thèse, 1999

II. Exemple de la rupture de barrage

II.2 Comparaison entre théorie, approches intégrées et simulation directe



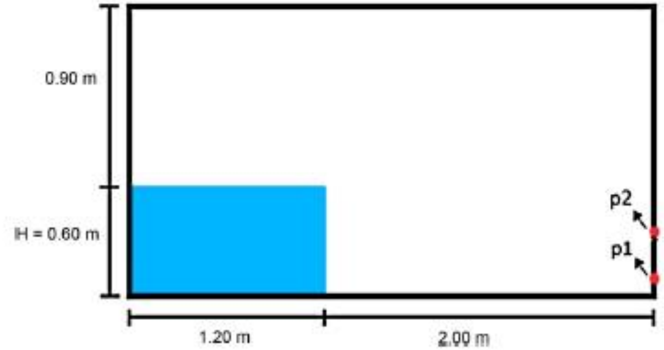
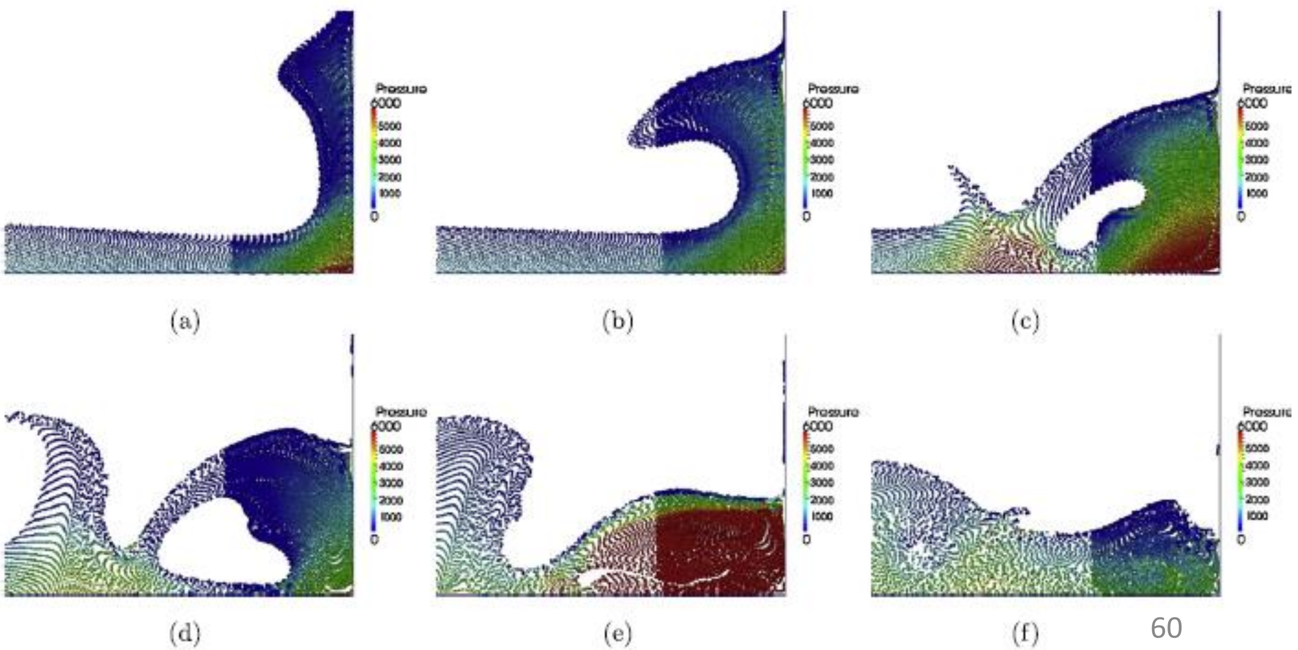


Fig. 8. 2D dam-break simulation sketch.

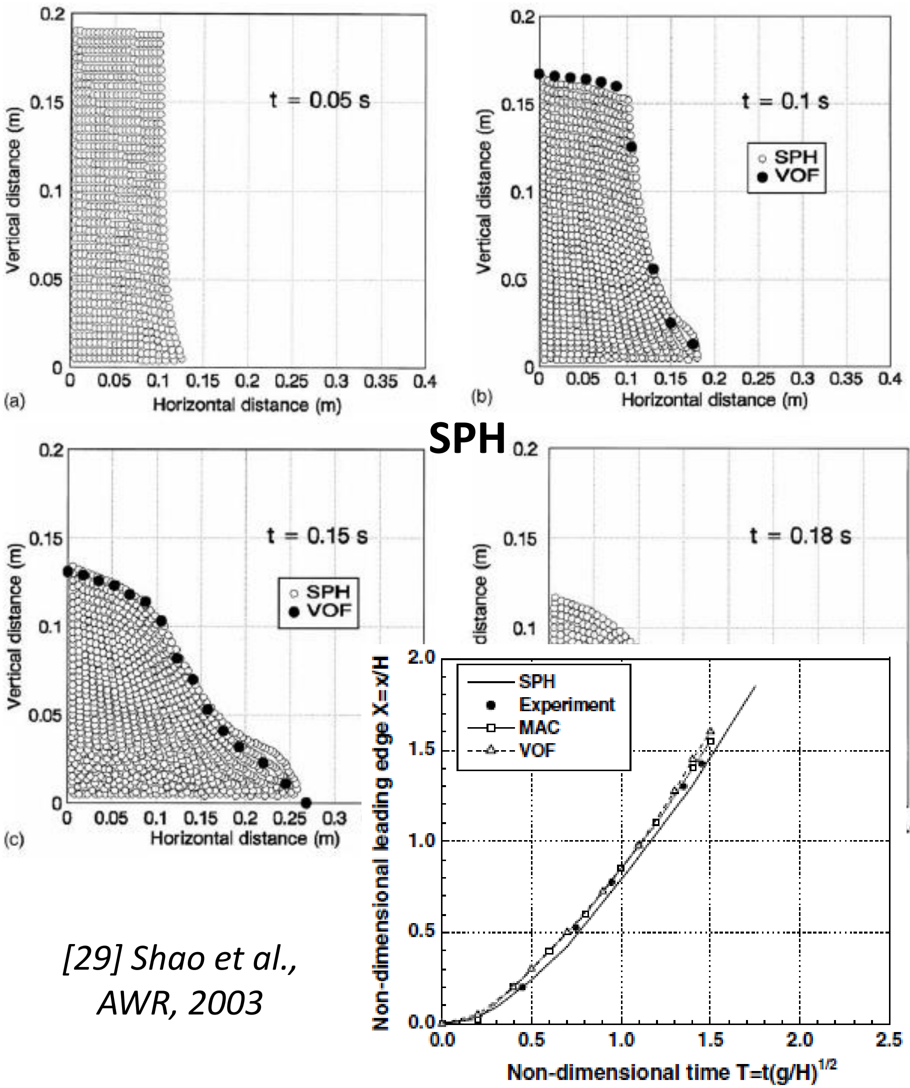
SPH



[27] Barcaroloa et al., JCP, 2014

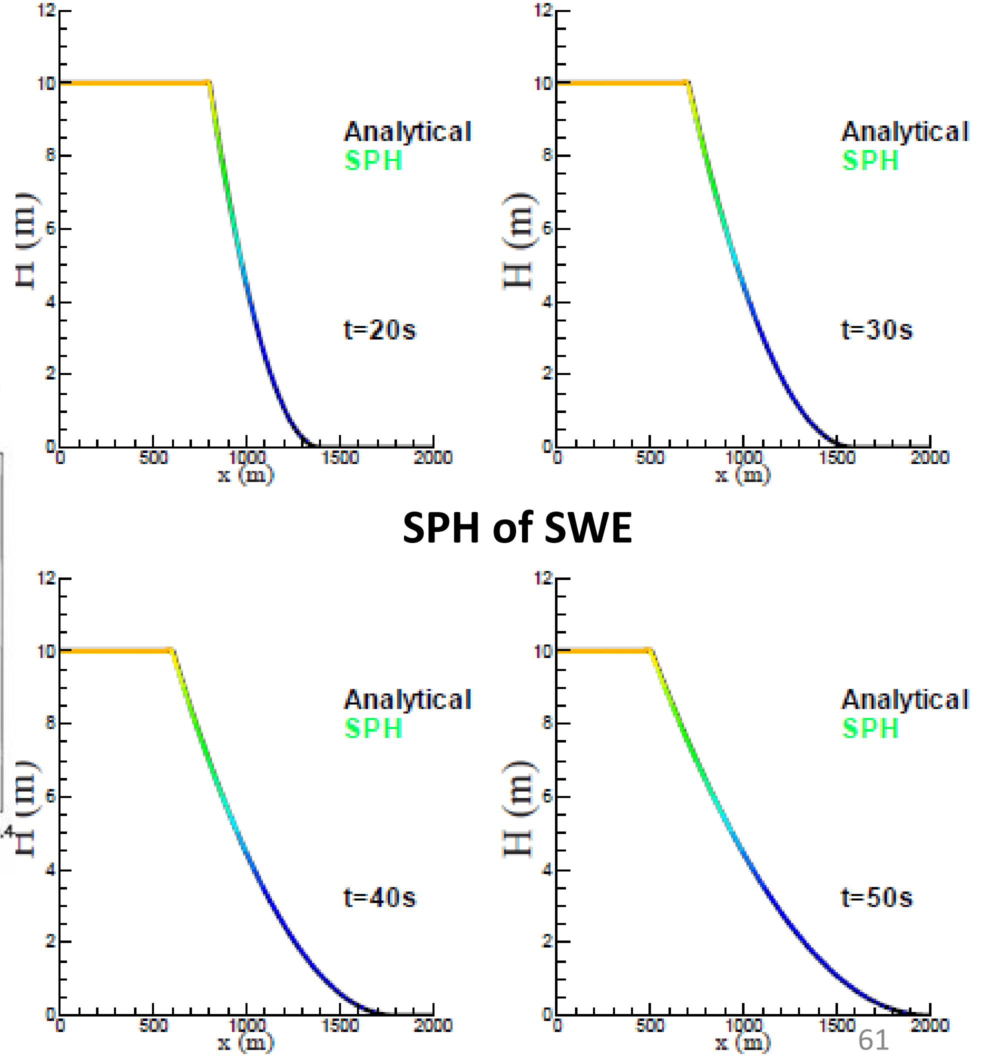
II. Exemple de la rupture de barrage

II.2 Comparaison entre théorie, approches intégrées et simulation directe



SPH

[29] Shao et al., AWR, 2003

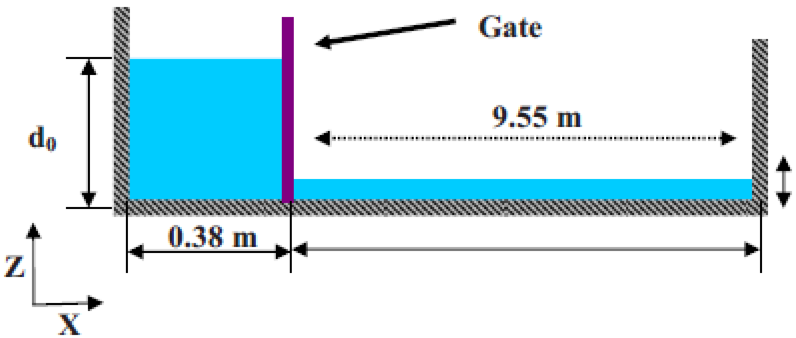


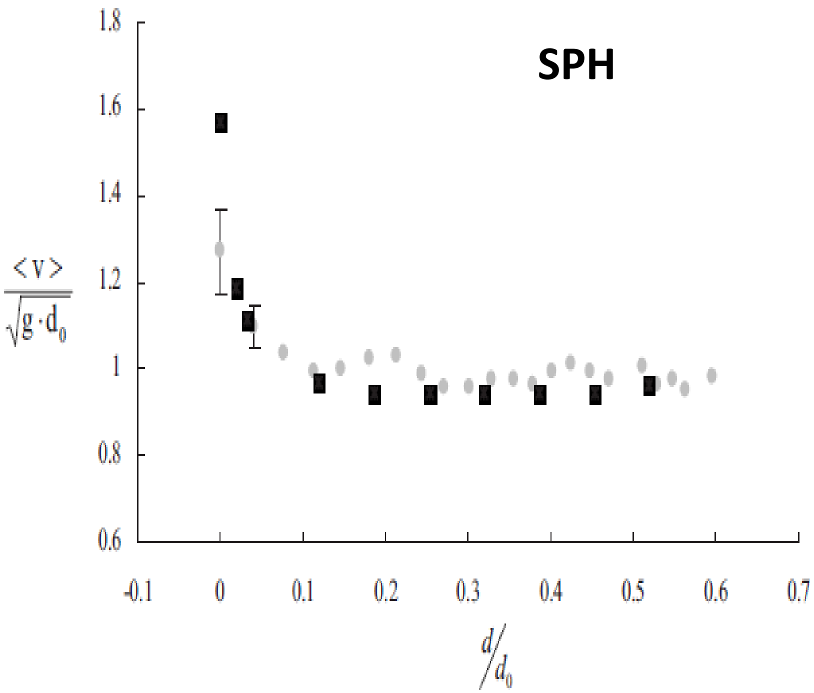
SPH of SWE

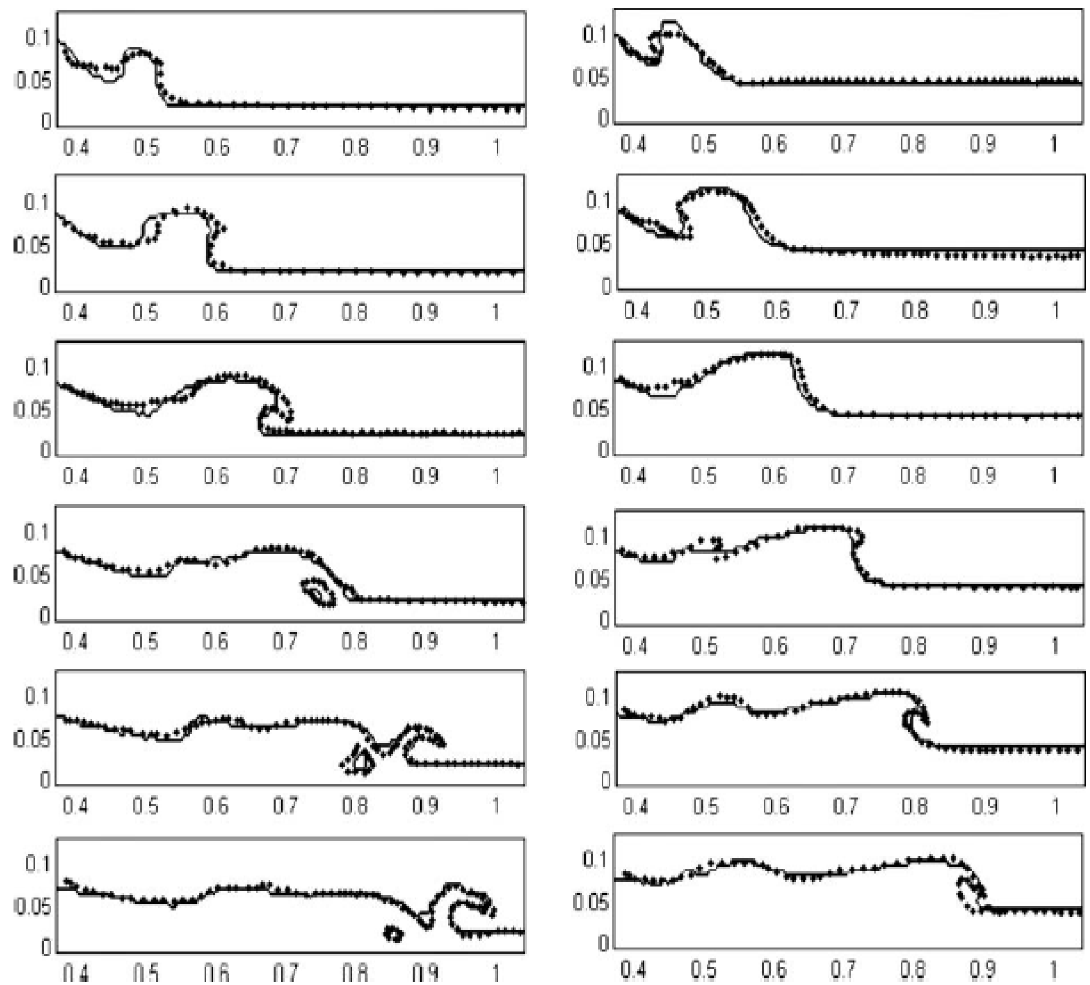
[28] Feffe et al., ICHD, 2008

II. Exemple de la rupture de barrage

II.2 Comparaison entre théorie, approches intégrées et simulation directe



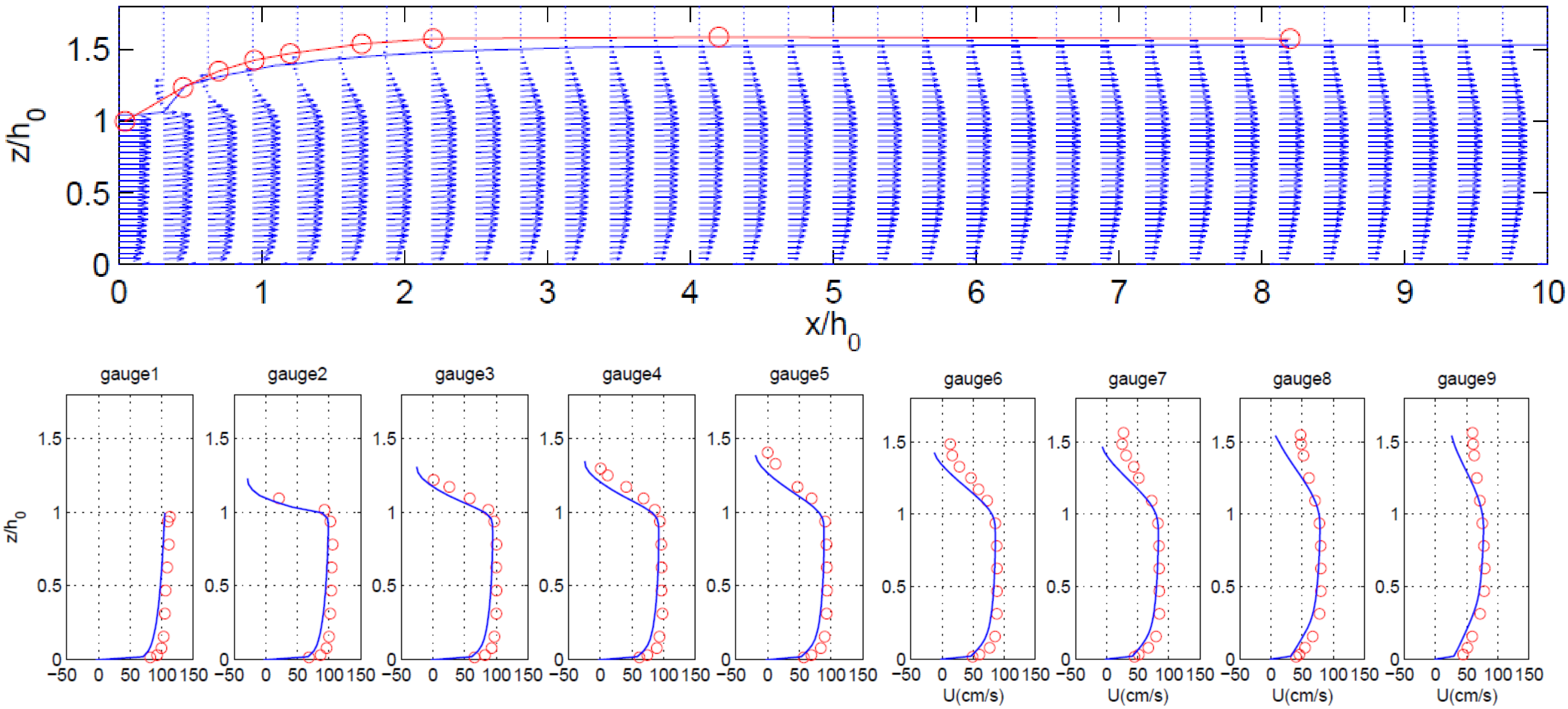




[30] Crespo et al., JWPCOE, 2008

III. Applications

III.1 Ressauts hydrauliques



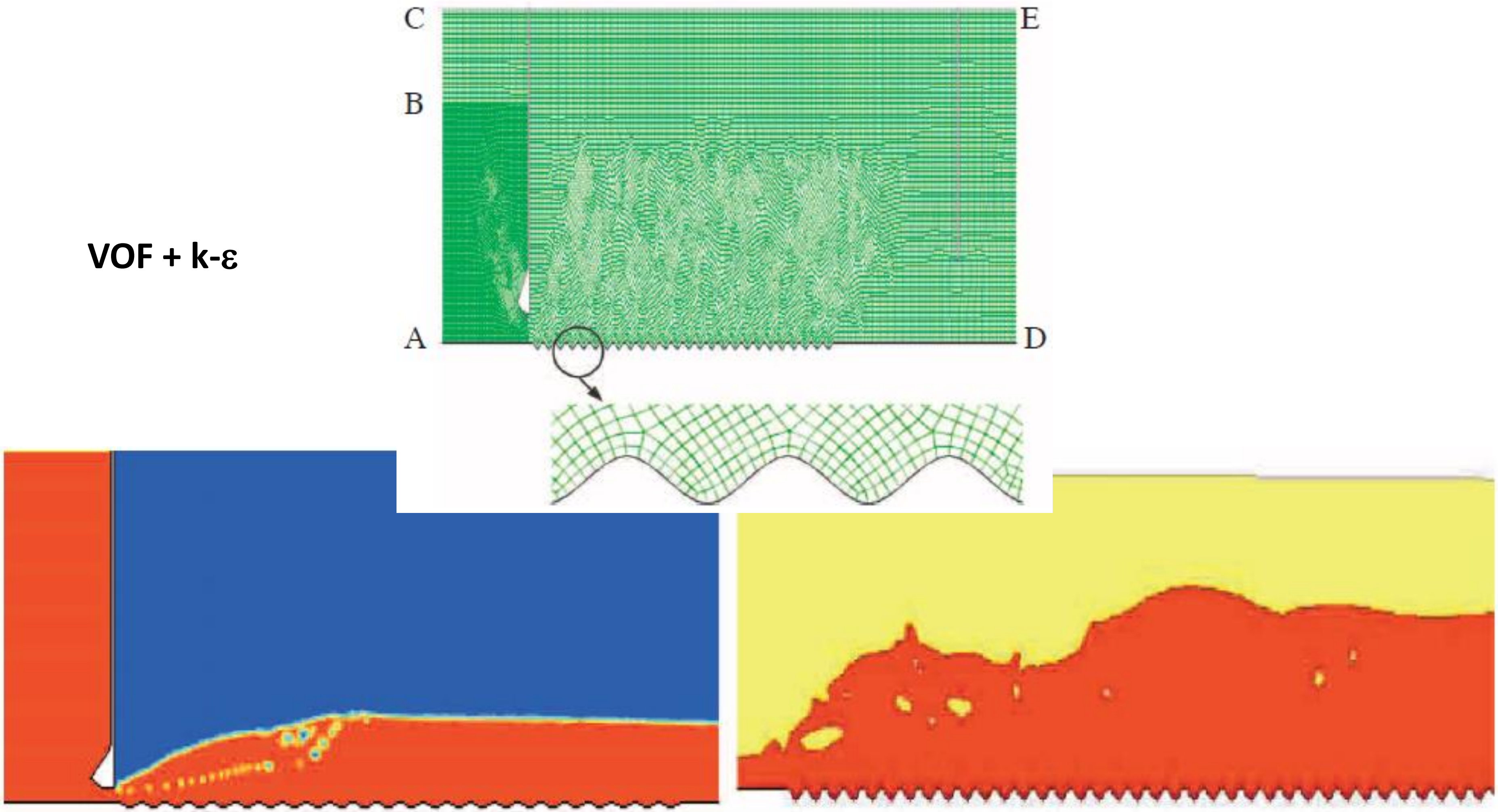
VOF + k-ε

[31] Zhao et al., EM, 2004

III. Applications

III.1 Ressauts hydrauliques

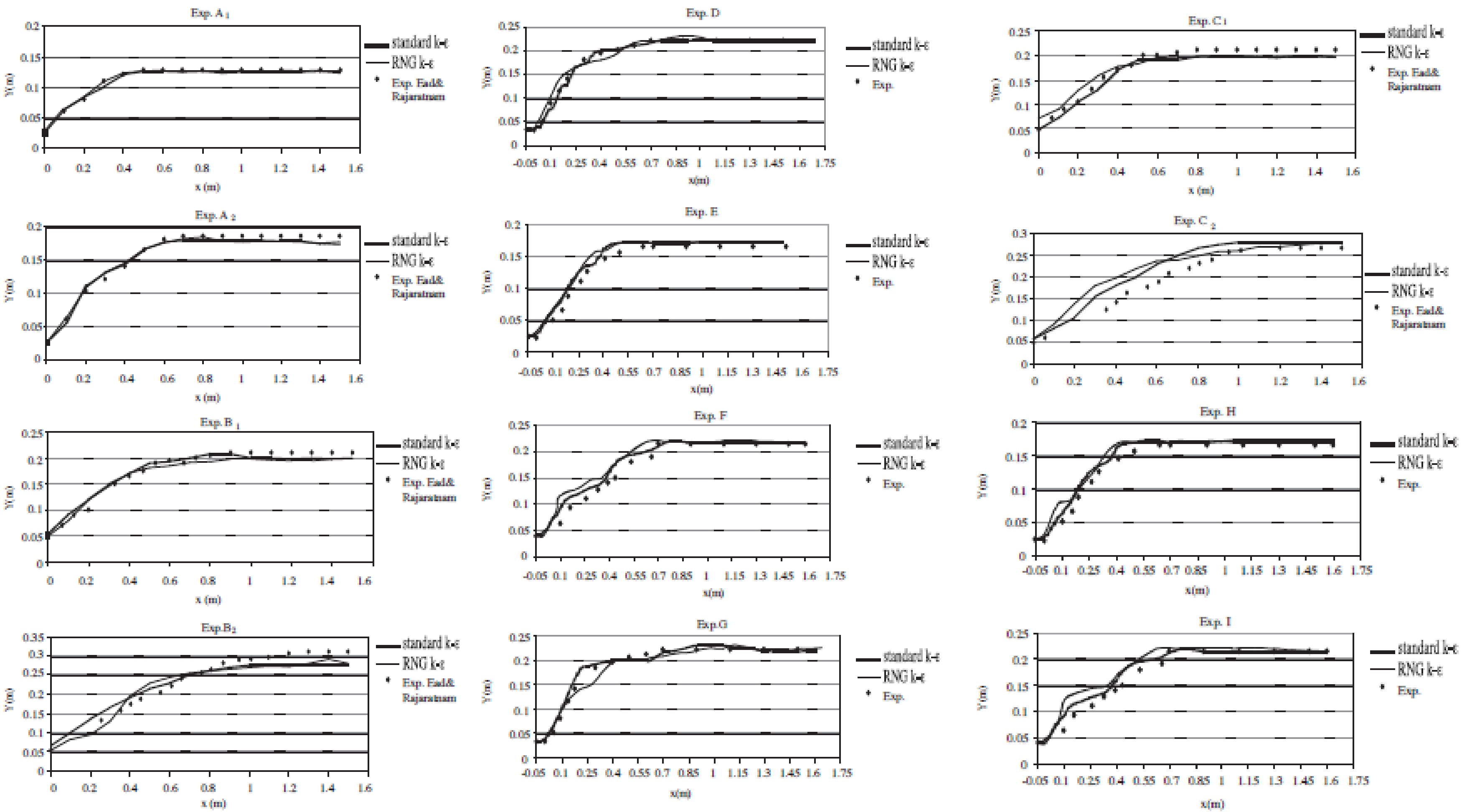
VOF + k-ε



[32] Abbaspour et al. , TJEES, 2009

III. Applications

III.1 Ressauts hydrauliques

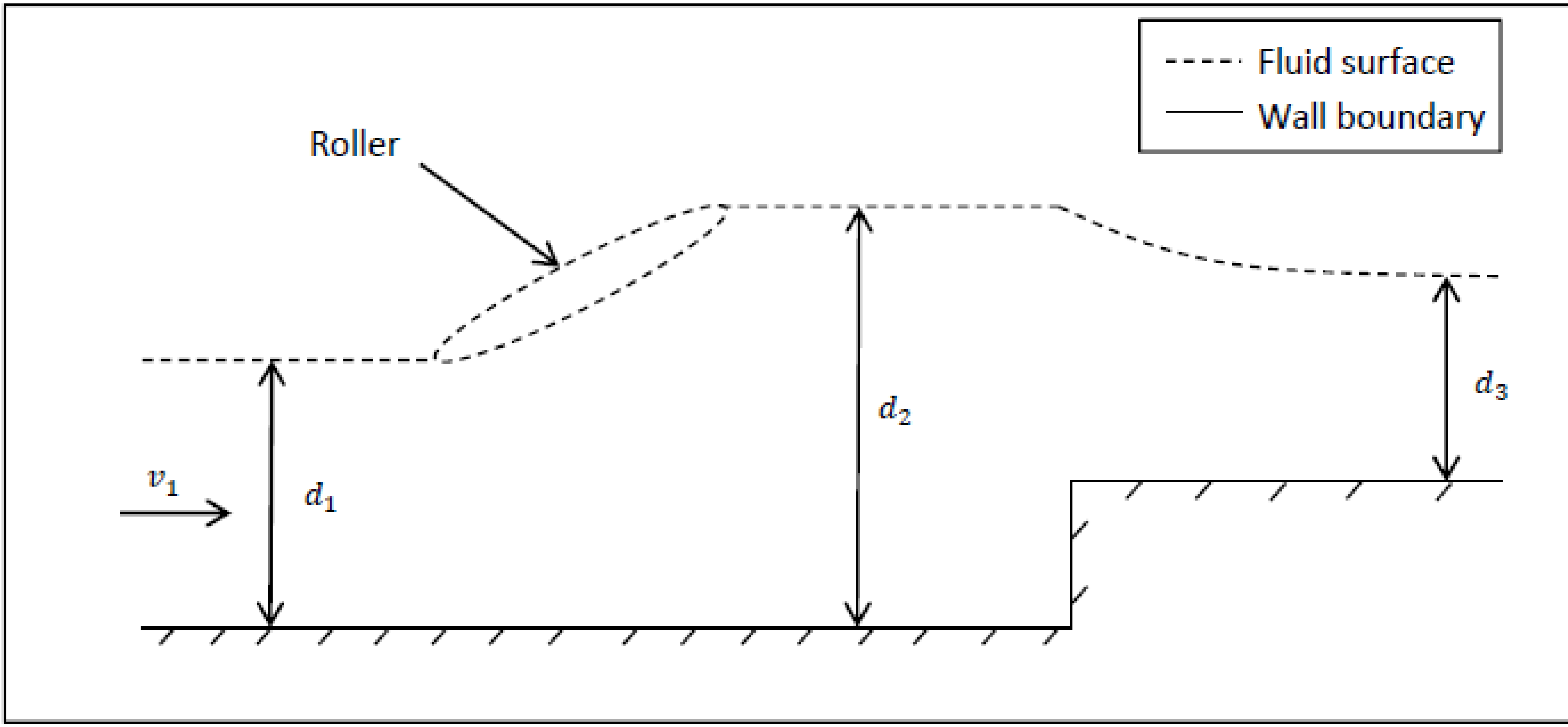


VOF + k-ε

[32] Abbaspour et al. , TJEES, 2009

III. Applications

III.1 Ressauts hydrauliques

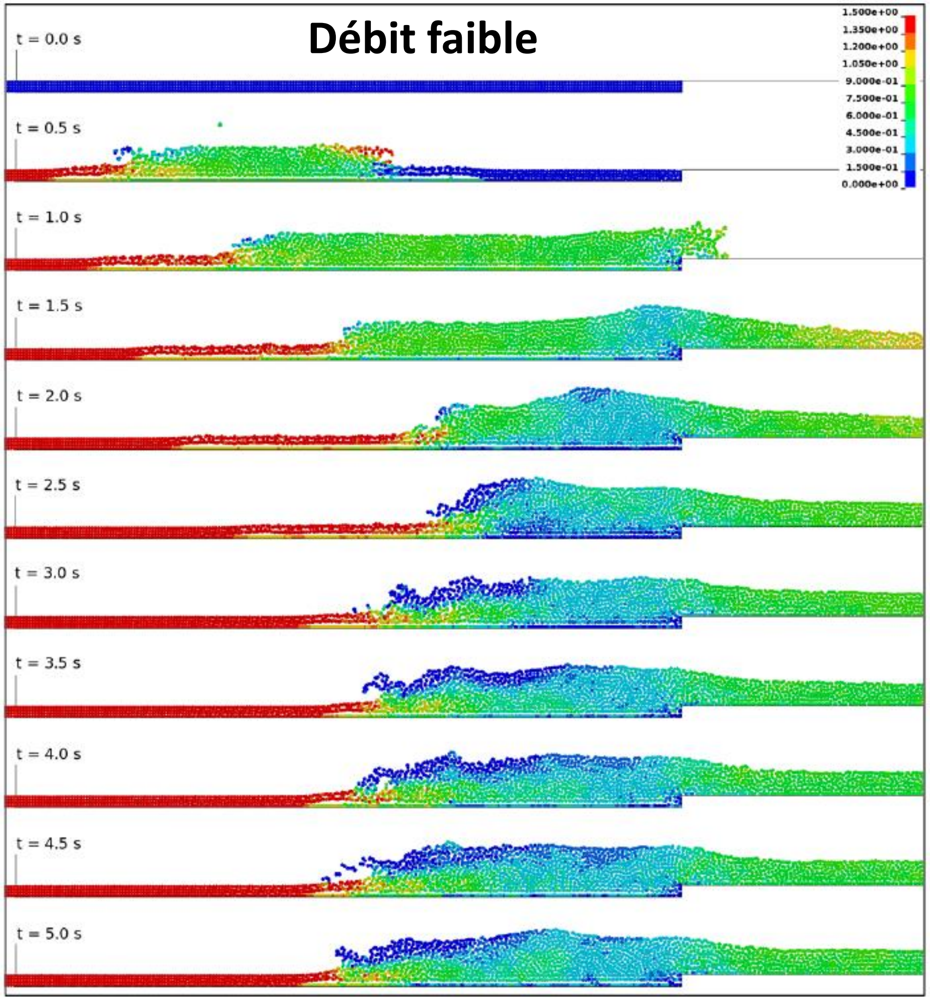


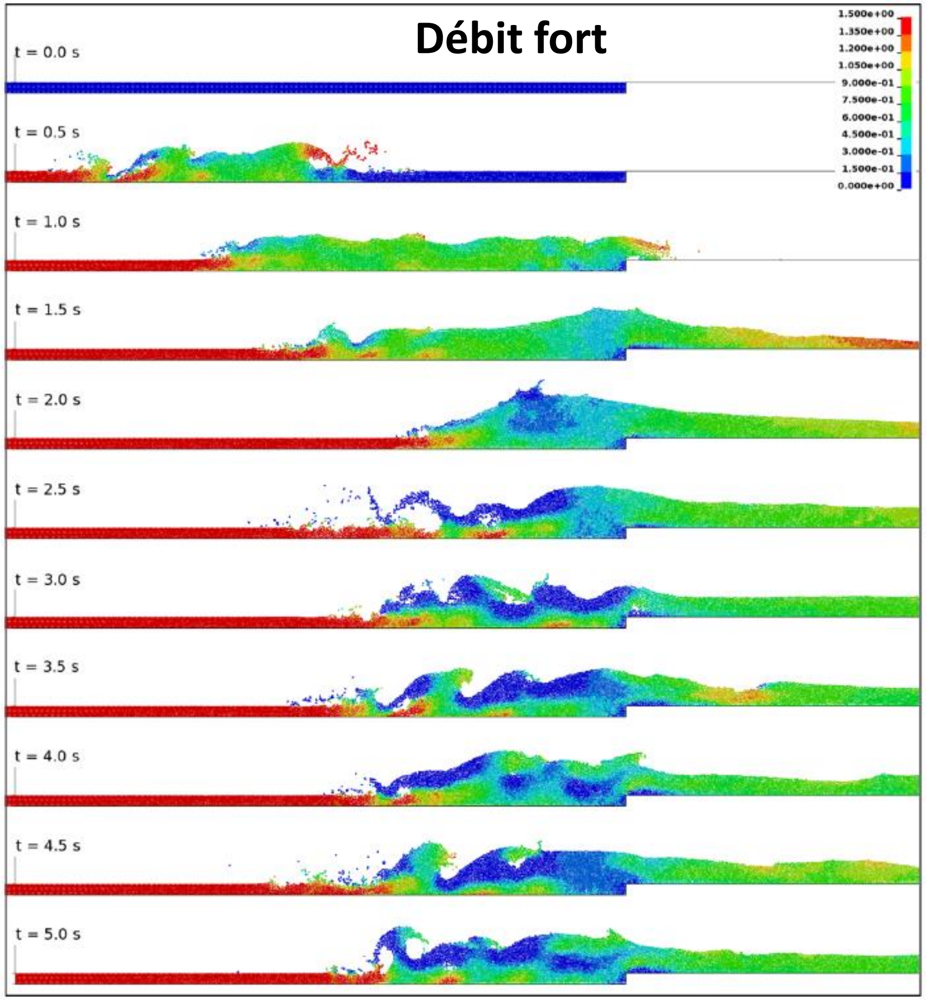
$$\frac{d_2}{d_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2}-1\right) \qquad Fr_1 = \frac{v_1}{\sqrt{gd_1}}$$

[33] Abbaspour et al. , TJEES, 2009

III. Applications

III.1 Ressauts hydrauliques





SPH

[33] Abbaspour et al. , TJEES, 2009

III. Applications

III.1 Ressauts hydrauliques

Table 2: The theoretically derived depth d_2 and numerical results obtained for the three cases coarse, mid and fine.

$time\ [s]$	$d_{2,theory}\ [m]$	$d_{2,c}\ [m]$	$d_{2,m}\ [m]$	$d_{2,f}\ [m]$
3.0	0.086	0.083	0.087	0.067
3.5	0.086	0.082	0.082	0.088
4.0	0.086	0.083	0.083	0.083
4.5	0.086	0.084	0.083	0.082
5.0	0.086	0.081	0.086	0.088

Table 3: The theoretically derived depth d_3 and numerical results obtained for the three cases coarse, mid and fine.

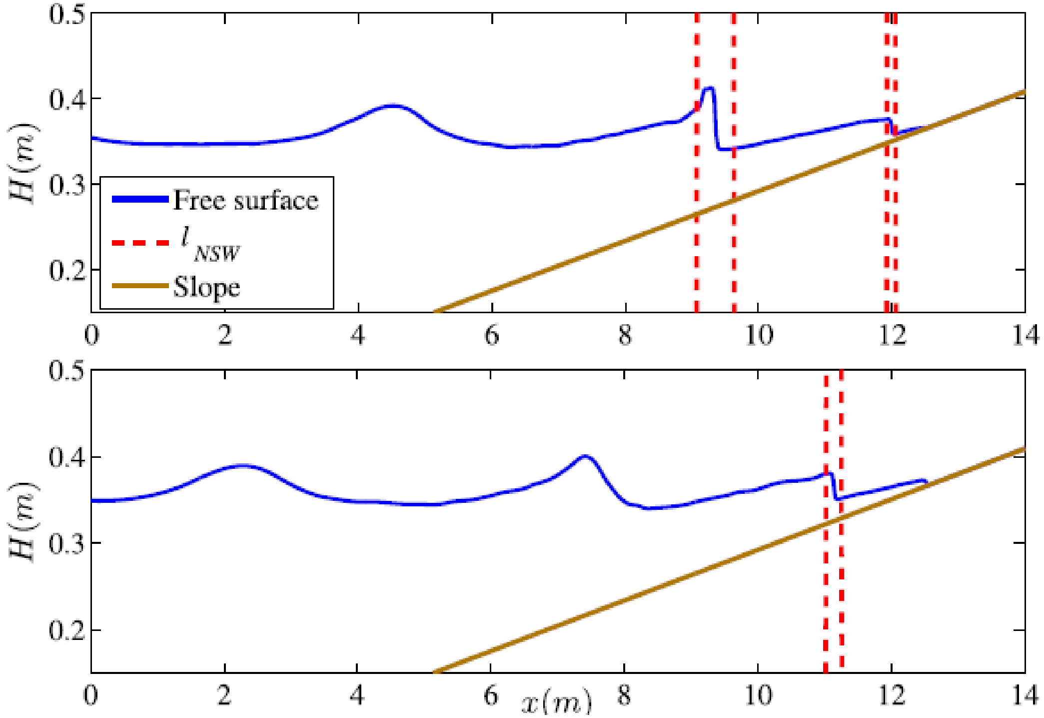
$time\ [s]$	$d_{3,theory}\ [m]$	$d_{3,c}\ [m]$	$d_{3,m}\ [m]$	$d_{3,f}\ [m]$
3.0	0.058	0.060	0.057	0.060
3.5	0.058	0.056	0.059	0.055
4.0	0.058	0.058	0.061	0.053
4.5	0.058	0.058	0.058	0.061
5.0	0.058	0.058	0.058	0.052

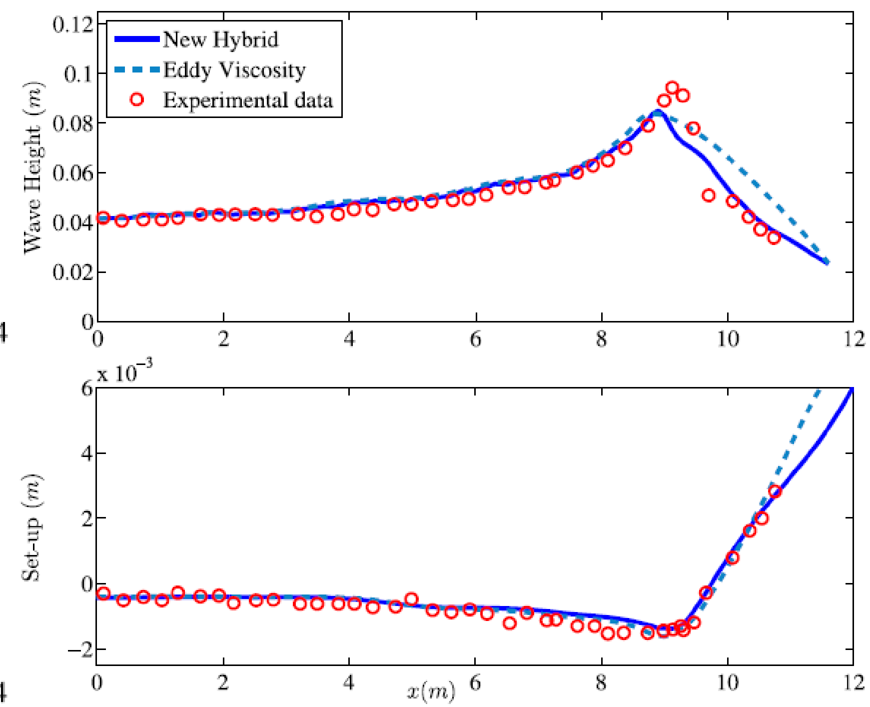
SPH – c=coarse grid, m=medium grid, f=fine grid

[33] Abbaspour et al. , TJEES, 2009

III. Applications

III.2 Déferlement de vagues





$$\partial_t \mathbf{U} + \nabla \cdot \mathcal{H}(\mathbf{U}^\star) = \mathbf{S}$$

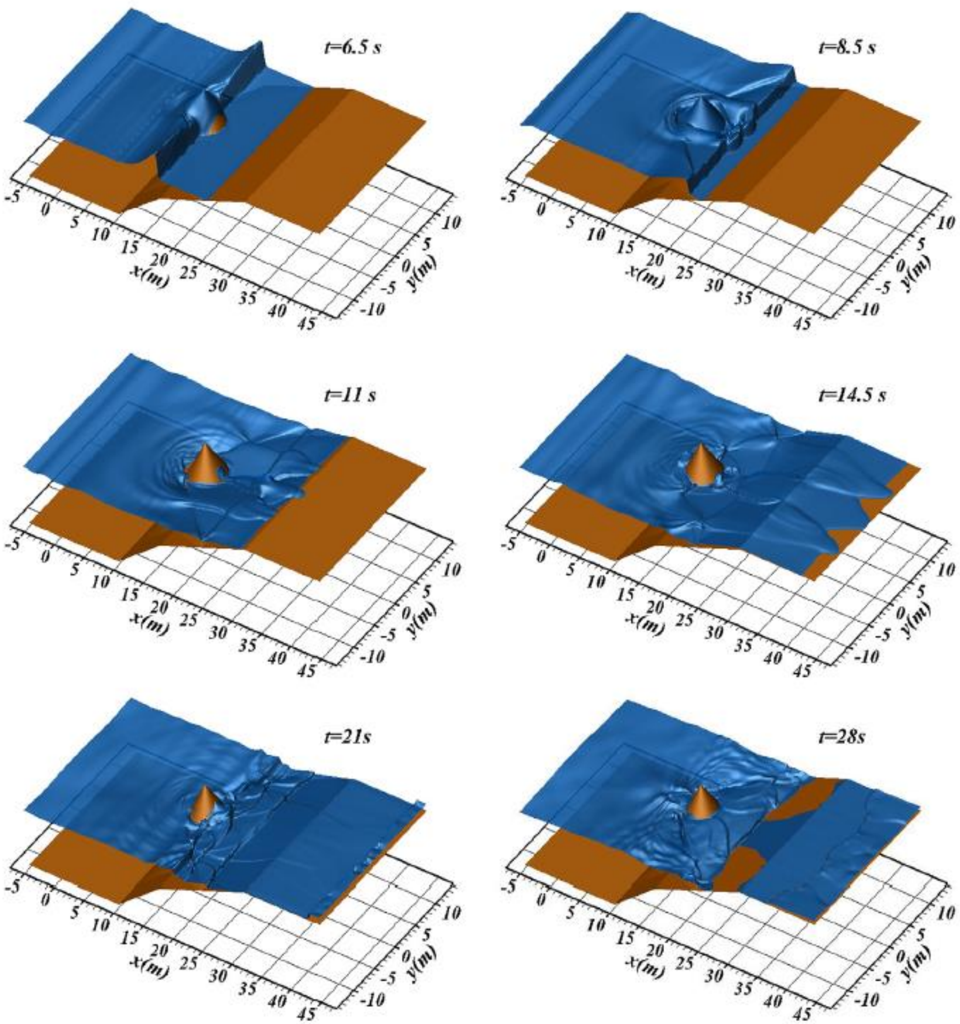
SWE – Boussinesq – Eddy viscosity model

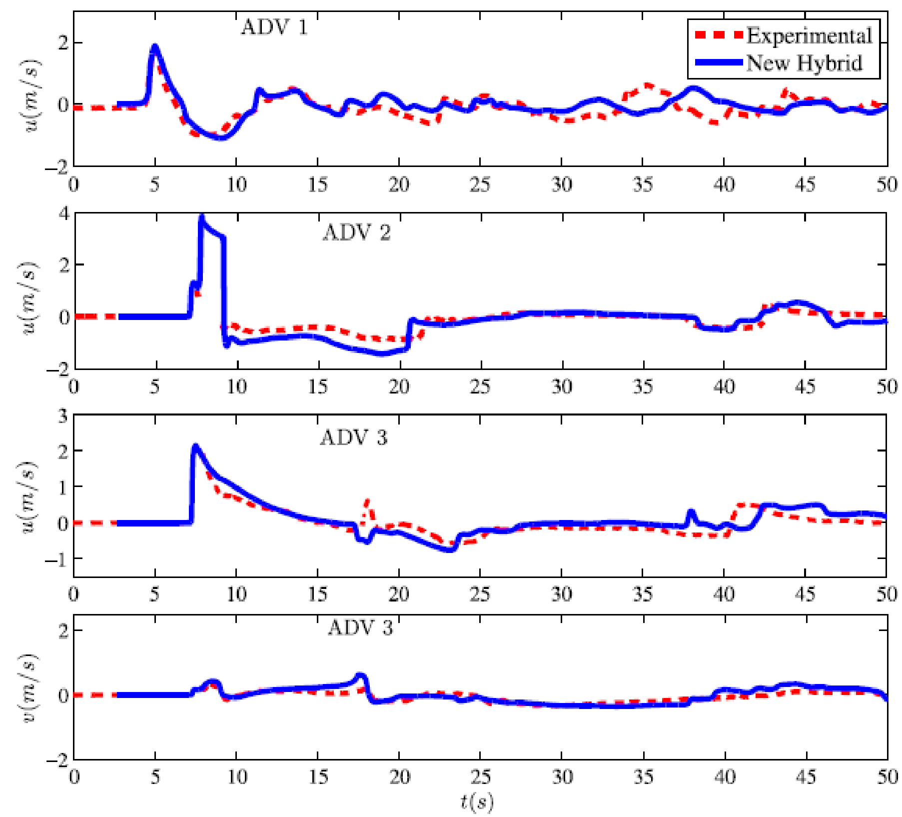
$$\mathbf{U} = \begin{bmatrix} H \\ P_1 \\ P_2 \end{bmatrix}, \qquad \mathbf{F} = \begin{bmatrix} Hu \\ Hu^2 + \frac{1}{2}gH^2 \\ Huv \end{bmatrix}, \qquad \mathbf{G} = \begin{bmatrix} Hv \\ Huv \\ Hv^2 + \frac{1}{2}gH^2 \end{bmatrix}$$

[34] Abbaspour et al. , TJEES, 2009

III. Applications

III.2 Déferlement de vagues





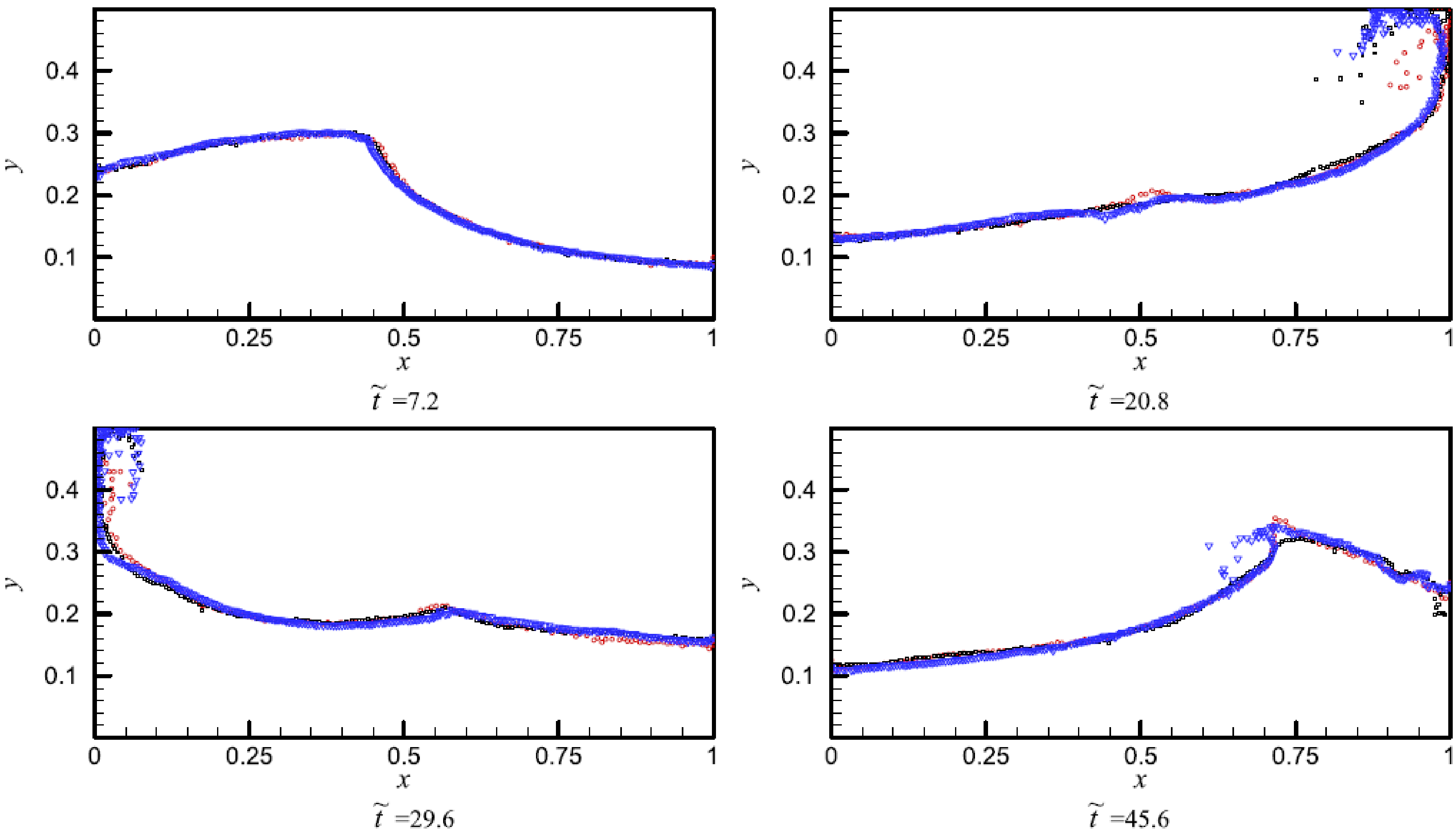
ADV measurements vs simulations

SWE – Boussinesq – Eddy viscosity model

[34] Abbaspour et al. , TJEES, 2009

III. Applications

III.2 Déferlement de vagues

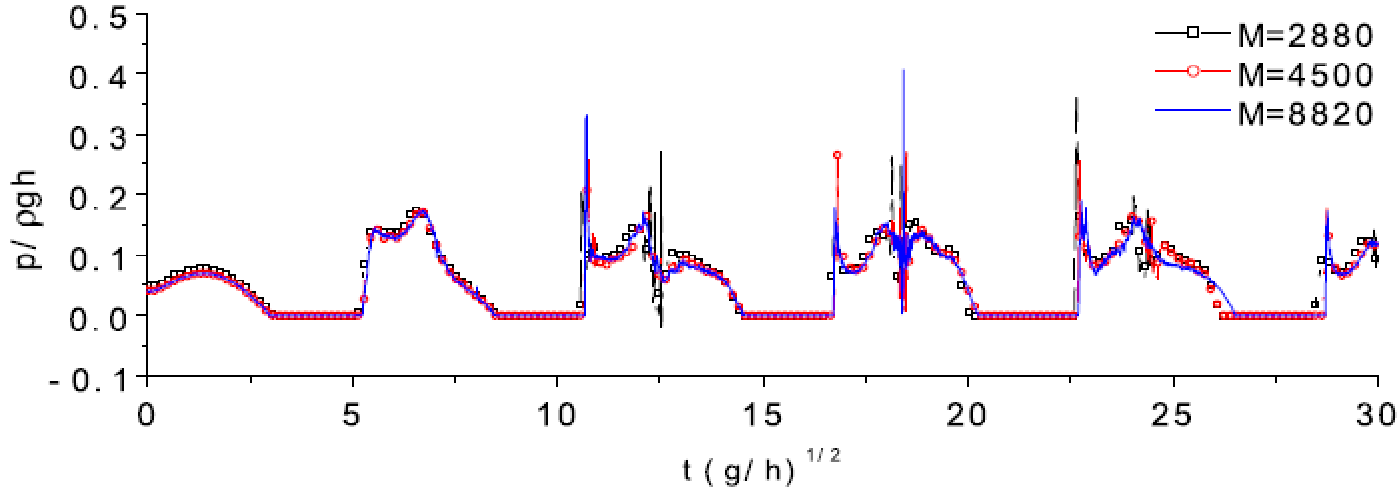


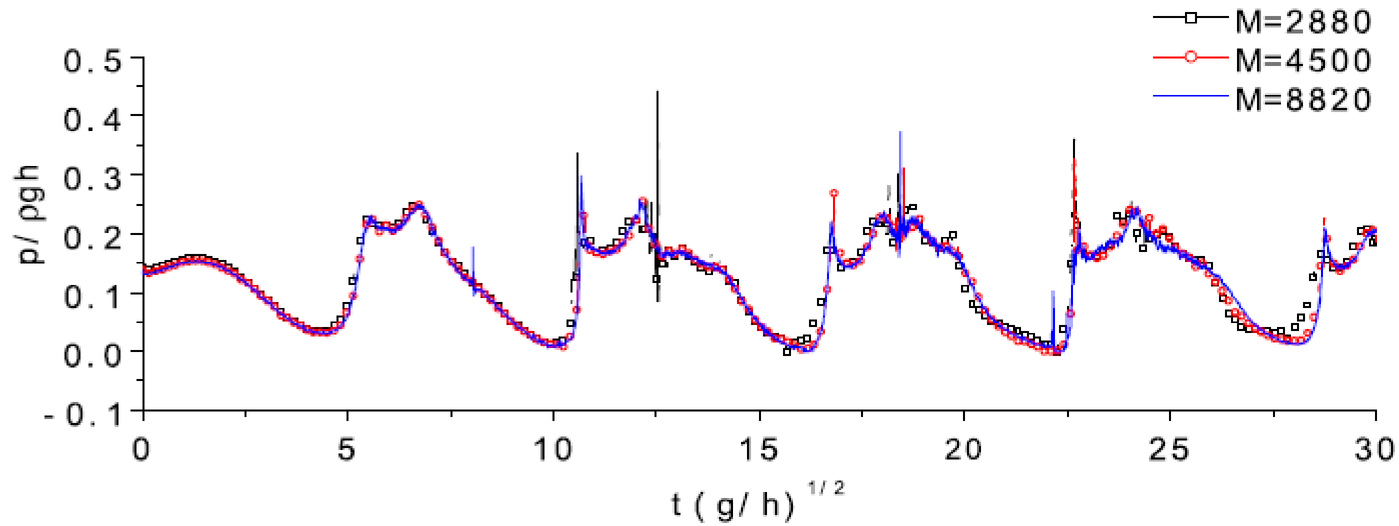
SPH

[36] Zheng et al., JCP, 2014

III. Applications

III.2 Déferlement de vagues



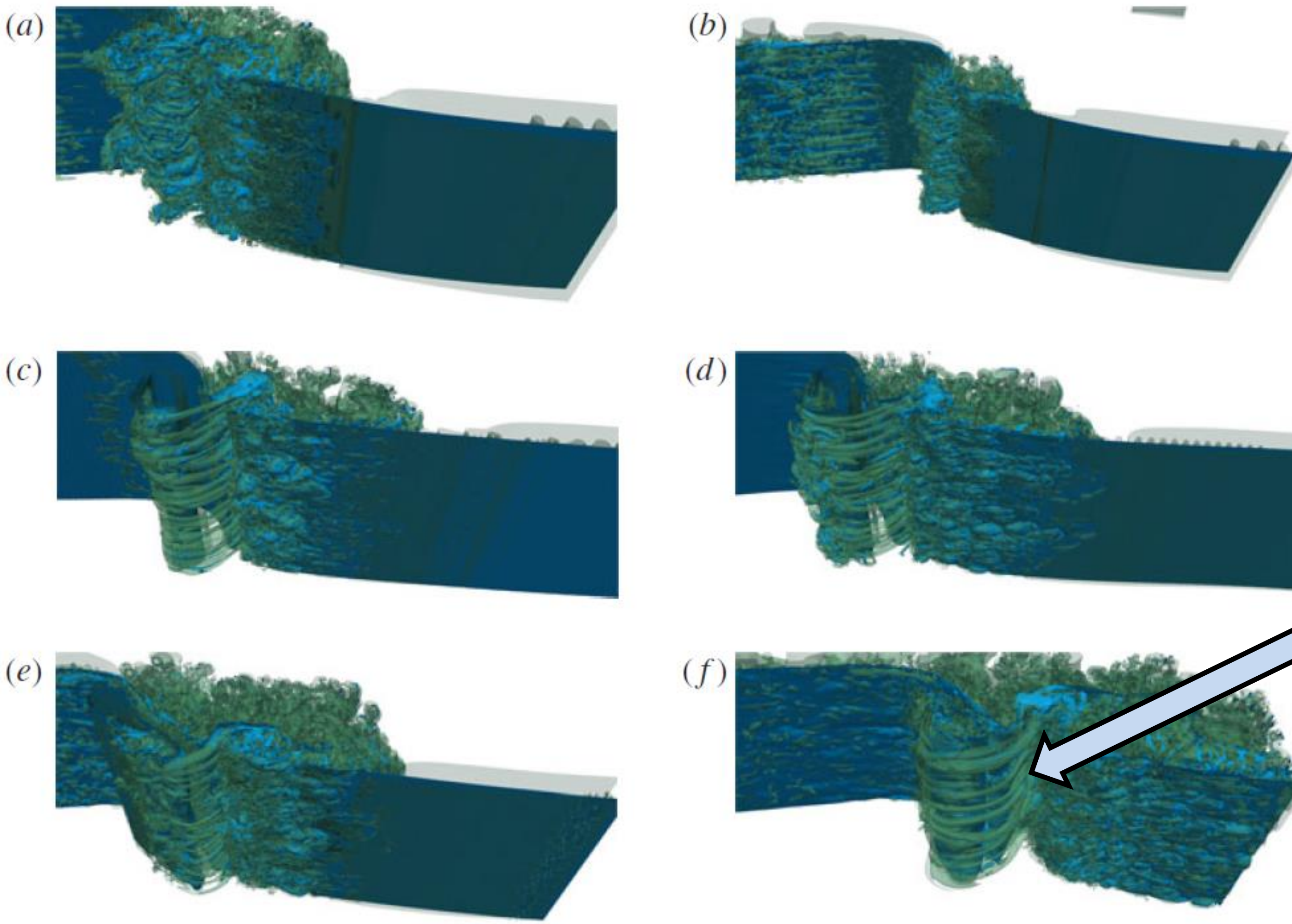


SPH

[36] Zheng et al., JCP, 2014

III. Applications

III.2 Déferlement de vagues



Aerated vortex filaments

Grids

$1024 \times 500 \times 200$

$1024 \times 500 \times 400$

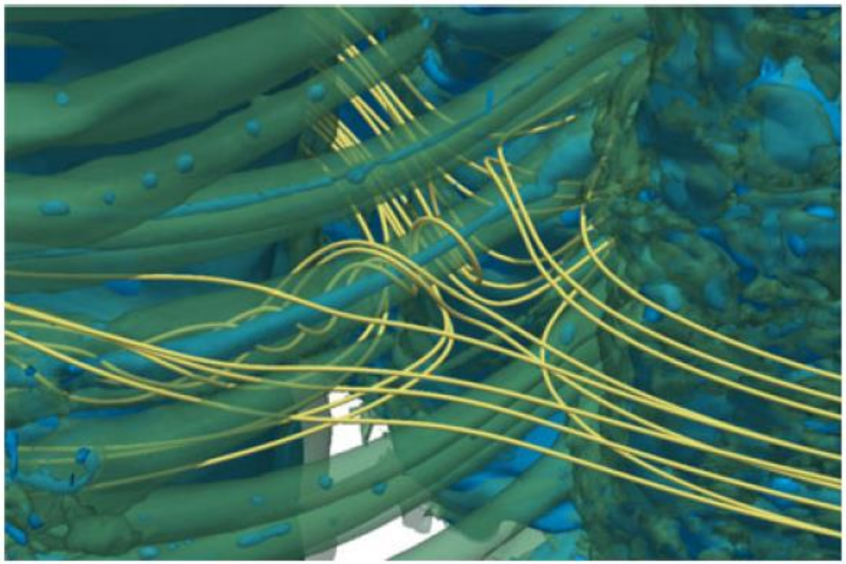
$2048 \times 600 \times 400$

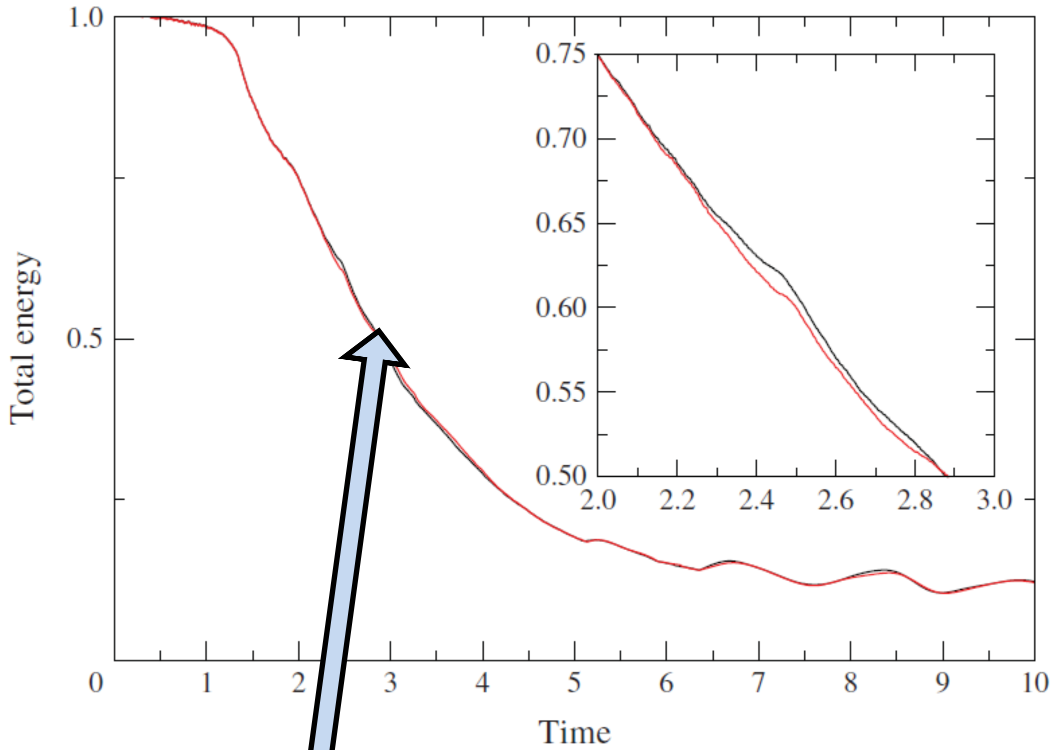
VOF - MUF

[37] Lubin et al., JFM, 2015

III. Applications

III.2 Déferlement de vagues





t^{-1} decay of kinetic energy – comparisons between fine and coarse grid

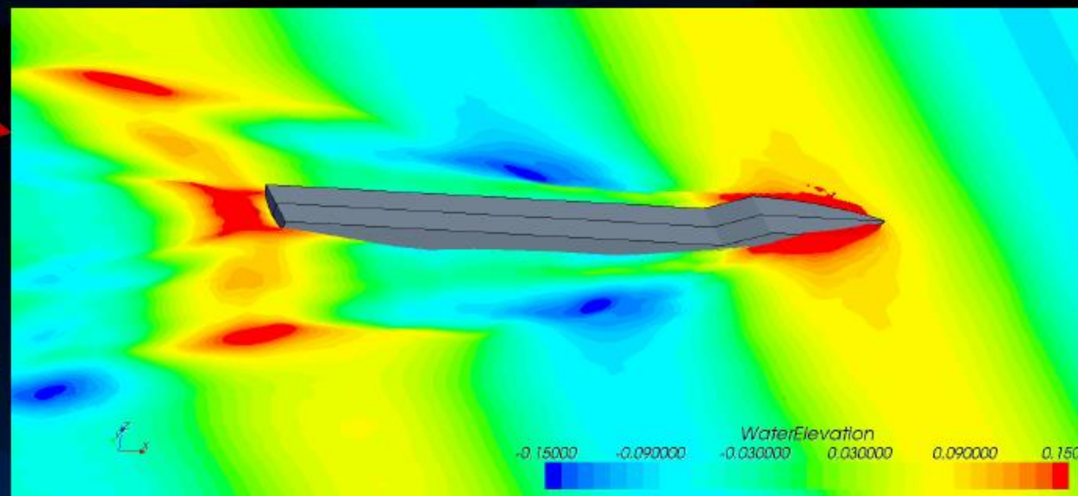
VOF - MUF

[37] Lubin et al., JFM, 2015

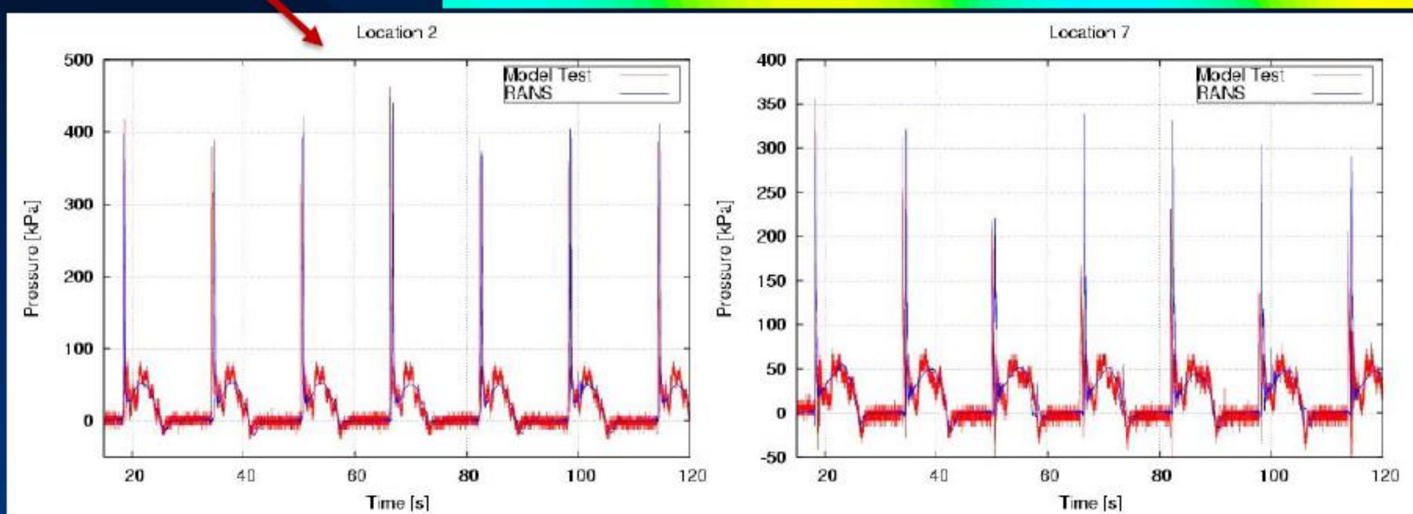
III. Applications

III.3 Impact de vagues sur des bateaux

Container ship in waves



Comparison of predicted and measured mean pressure over limited area at two bow locations (analysis by GL).

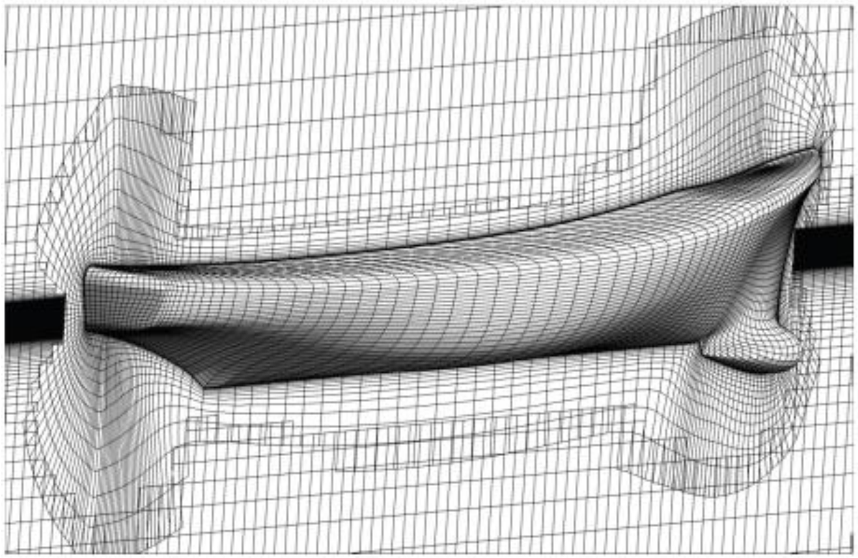


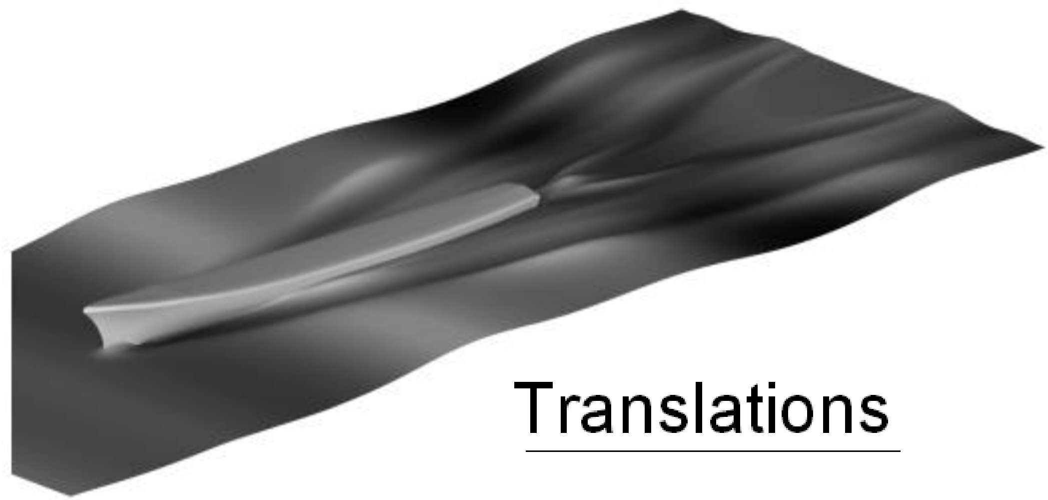
VOF – MUF - UNS

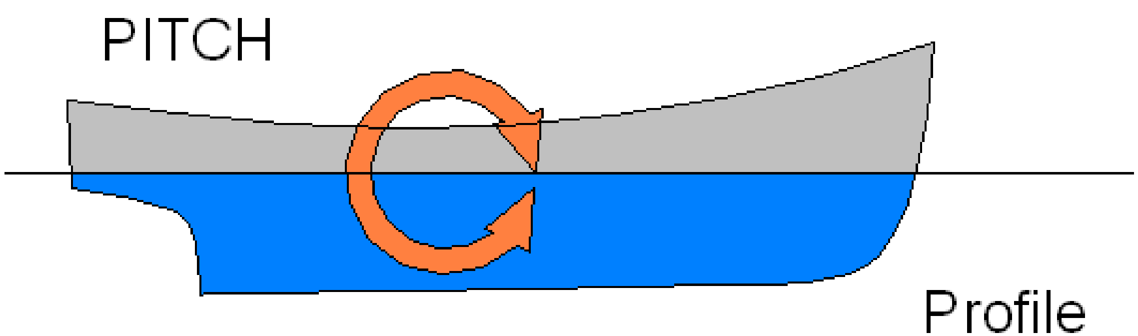
CD-ADAPCO – StarCCM+

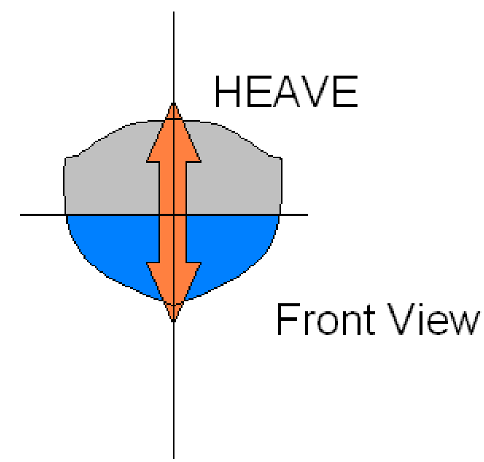
III. Applications

III.3 Impact de vagues sur des bateaux







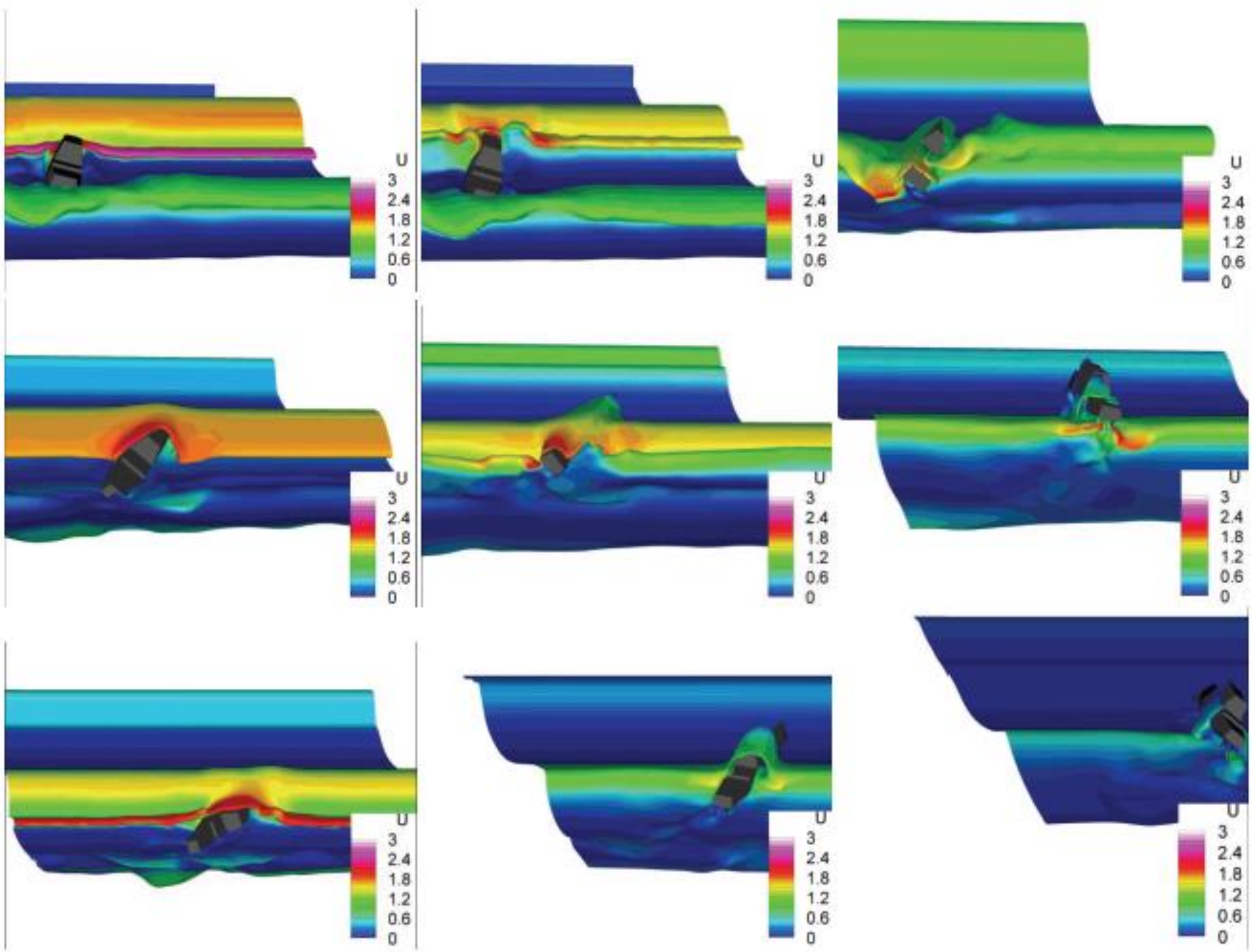


Level Set – MUF - UNS

[38] Mousaviraad, thèse, 2010

III. Applications

III.3 Impact de vagues sur des bateaux

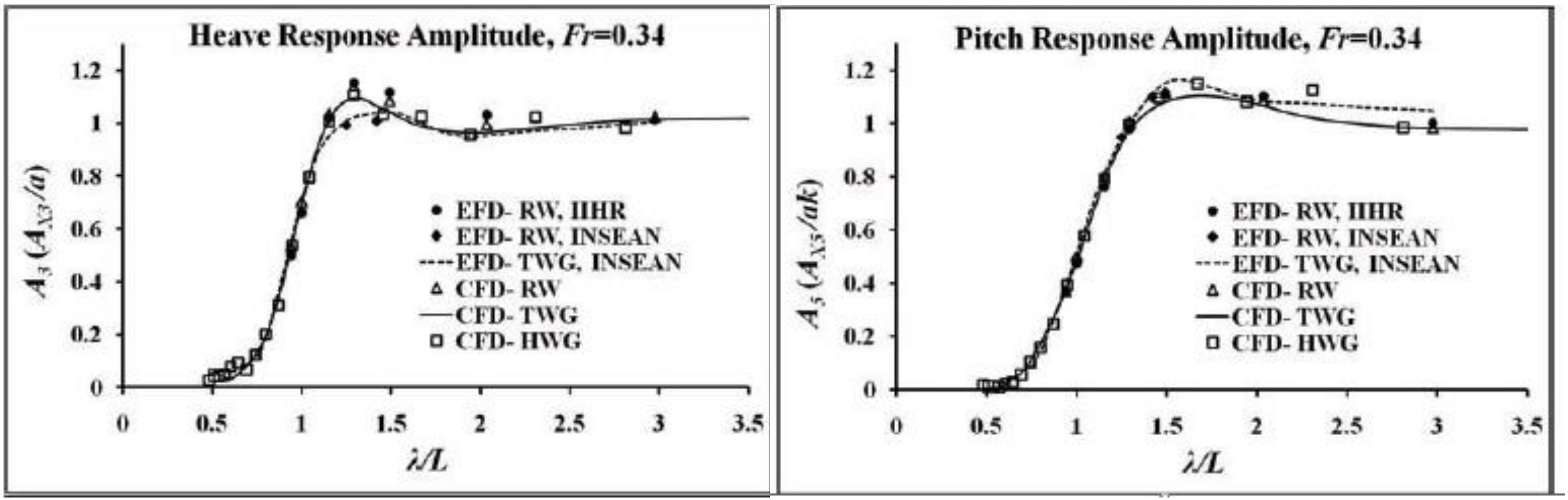


Level Set – MUF - UNS

[38] Mousaviraad, thèse, 2010

III. Applications

III.3 Impact de vagues sur des bateaux



EFD = experiments

RW = random waves

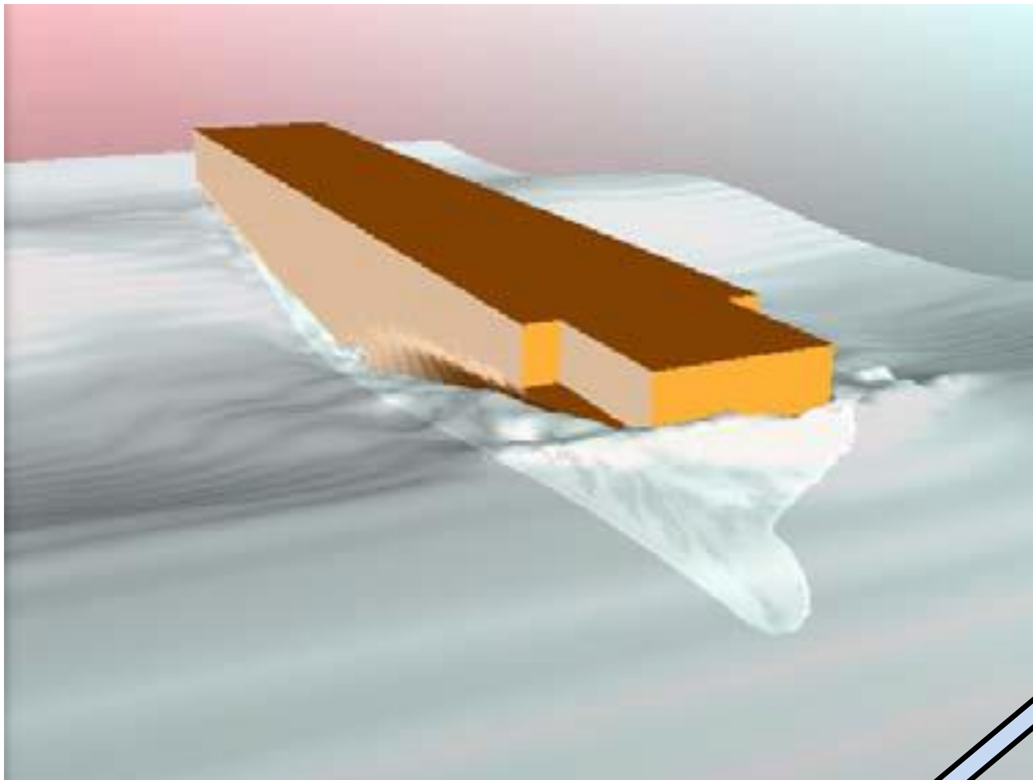
TWG = transient wave group

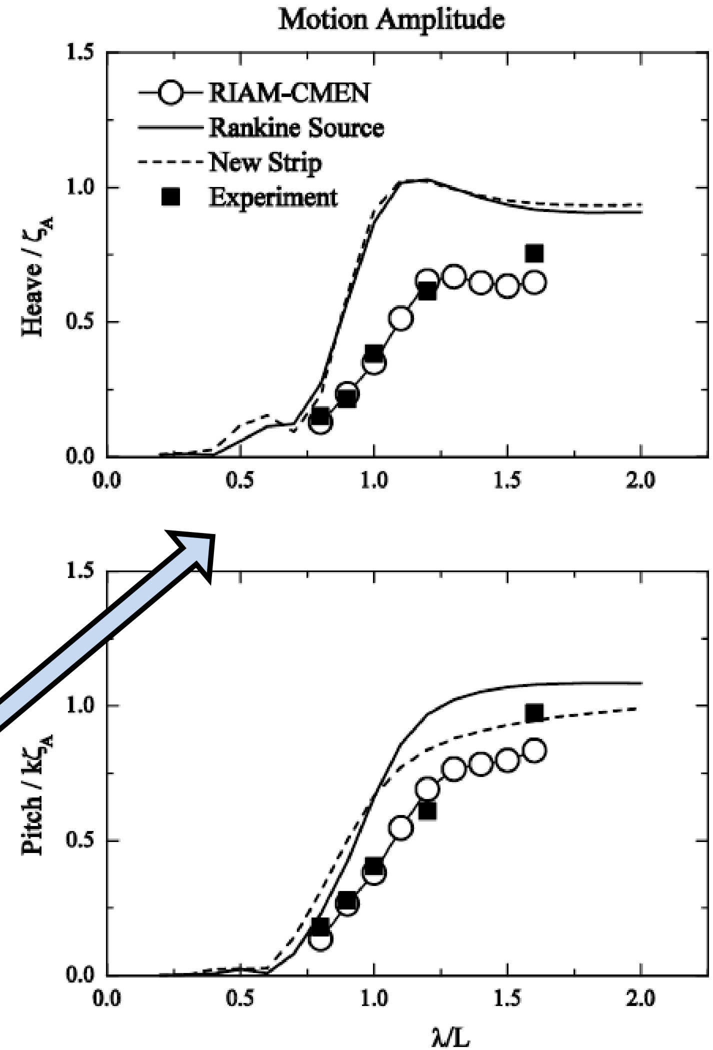
HWG = harmonic wave group

[38] Mousaviraad, thèse, 2010

III. Applications

III.3 Impact de vagues sur des bateaux





Frequency response to the wave induced motion

Comparison between VOF and SWE

VOF-CIP – MUF - IBM

[38] Hu, private com.

Synthèse

Modélisation des surfaces libres :

- **Lois d'échelle 10%** ↘
- **Approches intégrées 40%** →
- **Modélisation eulériennes locales MUF 20%** ↗
- **Modélisations lagrangiennes SPH 28%** ↘
- **DNS UNS/ALE 2%** →

Peu de comparaisons entre les différentes approches pour un même problème

Quel modèle pour quel problème :

- **Simulation réalistes très grande échelle (barrage, fleuve, océan, météo) => SWE**
- **Analyses locales, compréhension physique, estimation de coefficients pour SWE => MUF, SPH**
- **Validation, analyse rapide, ingénierie => lois d'échelle, SWE**
- **Résolution directe 3D, l'avenir mais difficultés numériques => DNS UNS/ALE**

Problèmes encore non résolus :

- **Modélisation de la turbulence => MUF + LES, viscosité dans SPH, modèles RANS dans SWE)**
- **Représentation réaliste et couplage de modèles => VOF + SWE + Lagrangien**

Représentation réaliste et couplage de modèles

[35] Zheng et al. , JCP, 2015

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